Paper 3 June 22

1) Smallest number is 4.301

2) $\begin{bmatrix} -4 \\ 8 \end{bmatrix} - \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -1 \\ 10 \end{bmatrix}$

3a) D is best fit for straight line

B has an outlier.

Points below LQ − 1.5 IQR
or above UQ + 1.5 IQR
are considered outliers.

4) First check your calculator is set correctly.
does $\sin 30° = 0.5$ ✓ check

so $\tan x = \frac{10}{4}$

so $\tan^{-1} 10/4 = 68.2°$

5) weekend 1 total = 5
weekend 2 total = 9

$\frac{9}{5} = 1.8$ so percent increase is 80%

6) $y = x^2 - 4x - 5$

We are not asked to but we could factorise $(x-5)(x+1)$

and observe the x-intersects really are at $x = 5$ and $x = -1$

The turning point is midway between these roots at $x = 2$

Finally by completing the square

$x^2 - 4x - 5 = (x-2)^2 - 9$

Back to the question though

a) Roots are 5 and −1
reading straight off graph

b) Substituting $x = 2$ into
$x^2 - 4x - 5$ gives
$2^2 - 8 - 5 = -9$
so turning point is $(2, -9)$

7)

0	1	2	3	4	5
10	13	16	19	22	25
	-3	3	3	3	3

so $f(n) = 3n + 10$

0	1	2	3	4	5	6	7	8
	2	4	8	16	32	64	128	256

so we can calculate directly
8th term is 256

so $3n + 10 = 256$
$3n = 246$
$n = 82$

8)

	Fridge	Freezer	Total
A	3 : $330 \times 3/5$	2 : $330 \times 2/5$	330
B	294	$294 \times 3/7$	

so we have

A	198	132*	330
B	294	126	420

Grace buys fridge/freezer A.

9) Tom's speed is $\frac{200}{24} = 8\frac{1}{3}$ m/sec.

Adil speed is $28.8 \frac{\text{km}}{\text{hr}} \times \frac{1\text{hr}}{60\text{min}} \times \frac{1\text{min}}{60\text{sec}} \times \frac{1000}{1}$

$= 8$ m/sec.

So Tom will win the race.

This method is known as unit conversion.

We write the original value together with the units clearly stated and then multiply through by the converting factors which have value 1

i.e. $\frac{1\text{hr}}{60\text{min}}$ and $\frac{1\text{min}}{60\text{sec}}$ and $\frac{1000\text{m}}{1\text{km}}$

and we carefully cancel out the units as well as calculating the final conversion value.

10) Range is $3.55 \leq m < 3.65$

This question is a bit deep. Note the examiner wants you to select the 4th option where there is the $<$ sign not the $\leq$ sign.

The bottom line is if we add together the two values and halve we should recover the original value.

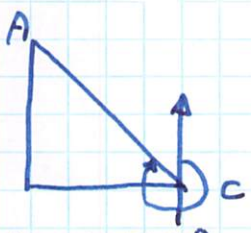
$(3.55 + 3.65) \div 2 = 3.6$.

11) $70°$ $110°$ $120°$ $50°$
angles in quadrilateral.

Note $70 + 110° = 180°$ so we have parallel lines.

So the quadrilateral is a TRAPEZIUM

12)



Bearing is 315°
Personally I don't even like the ° sign and would write

A from C is bearing 315.

{ To find the bearing of C from A we ± 180 }

12b) I think Mia has measured bearing O from E which could well be 068.

We want E from O which is $068 + 180 = 248$

13) $\sqrt{5}a + \sqrt{5}a$

$= 2\sqrt{5}a$

Think of $\sqrt{5}a$ as a "thing in its own right" so in algebra we have $x + x = 2x$.

14) This is a boxplot

The boxplot representation was invented by John Tukey and published in 1970 – that is relatively recently as most GCSE Maths was known to the ancient Greeks.

B. The tails are given as £3 and £26

Lower Quartile = £11
Median is A+2 = £14

IQR_A is $17 - 9 = 8$
IQR_B is $8 \times 1\frac{1}{2} = 12$

So we now have all the data points to draw the boxplot.

A boxplot summarises 5 data points

Lowest, LQ, Median, UQ, Highest and we can immediately read off IQR.

15)

$$278 \frac{\text{people}}{m^2} = \frac{158460 \text{ people}}{\text{area}}$$

$$\text{so area} = \frac{158460}{278} \text{ km}^2$$

So if population increases to 168720

$$\text{new population density} = \frac{168720 \times 278}{158460}$$

$$= 296 \frac{\text{people}}{\text{km}^2}$$

16) 3cm represents 1500 metres.

Virat's estimate of volume is $\pi r^2 d$

$= \pi \, 1500^2 \times 17$

$= 38\,250\,000\,\pi \text{ m}^3$

$= 120\,000\,000 \text{ m}^3$
to 2 sig. fig.

b) the actual depth is less the area is greater

So the actual volume could be greater or lesser than Virat's estimate.

17) Characters have choice of

8 faces 4 bodies 5 hairstyles

so number different characters are

$8 \times 4 \times 5 = 160$

b) P(two characters identical)

$\frac{1}{8} \times \frac{1}{4} \times \frac{1}{5} = \frac{1}{160}$

18) There are three "keys" to this.

1) Spot the StarTrek symbol
2) Radii form isoceles triangles.
3) Alternate angle theorem

1) gives $\angle BOC = 56°$
2) gives $\angle OBC = \angle OCB = 62°$
3) gives $\angle ABC = 83°$

so $\angle ABO = 83 - 62 = 21°$

Finally in $\triangle ABC$

$\angle ACO = 180 - 28 - 83 - 62$

$= 7°$

Therefore ABO : ACO is 21 : 7

$\rightarrow$ 3 : 1

19) Area $= x(x+1) - 3 \times 5$

$= 75$

$x^2 + x - 15 = 75$

$x^2 + x - 90 = 0$

b) $(x + 10)(x - 9) = 0$

so $x = 9$

and we discard the -ve value

20) $\frac{2496.96}{2448} = 1.02$

so interest rate is 2%

$2448 \times 1.02^4 = £2649.29$

21. By sine rule

$\frac{\sin x}{17} = \frac{\sin 64}{23}$

$x = \sin^{-1}\left(\frac{17 \sin 64}{23}\right)$

Using a calculator it is safest to enter

$17 \sin 64 = Ans$

$Ans / 23 = Ans$

$\sin^{-1} Ans = 41.63°$

22) $f(x) = 3x$

$g(x) = x^2$

so $fg(x)$ means $f\{g(x)\}$

but we don't bother with the {}

$fg(x) = 3x^2$

(lots of students will write $(3x)^2$...
why is this wrong)

23) $y = x^2 + 7x - c$

$y = 3x + d$

A solution is at $x = 5$ so we have

$25 + 35 - c = 15 + d$

so $c + d = 25 + 35 - 15$

$= 45$

24) we have sketches of $y = k^x$ ①

and $y = x^2 + c$ ②

Now for ① we have (4, 81)

so $81 = k^4$

so $k = 3$

The functions ① and ② intersect at (3, r)

so we substitute $x = 2$ and equate

$3^2 = 9$

and that now lets us deduce c in ②

$9 = 2^2 + c$

$c = 5$

and finally we use (r, 43.44) to produce

$43.44 = r^2 + 5$

$r = 6.2$

Nothing difficult about this, but you do need to think through 4 stages clearly

25) Examining the histogram we have the areas of the 4 blocks as

① = 0·6×10
② = 4×5
③ = 5×7·6
④ = 10×1·6

Total = 80

and that is the number checked so we fully understand the histogram.

So next we draw a vertical line at mass = 122 and calculate the area

Area = ① + 2×4
= 6 + 8
= 14

So making 28000 tubes we estimate

$\frac{14}{80} \times 28000 = 4900$ tubes

Addendum.
The question could have asked us to estimate LQ mean UQ

To do this we divide areas into 4 blocks and draw vertical lines at LQ a Median b UQ c.

$\frac{80}{4}$ lower quartile 20 median 40 upper quartile 60

LQ occurs in ①
Median occurs in ②
UQ occurs in ③

So LQ: $6 + 4a = 20$
$a = 3\frac{1}{2}$
so LQ $120 + 3·5 = 123·5$

Median $6 + 20 + 7·6b = 40$
$b = 1·8$
so Med $125 + 1·8 = 126·8$

UQ: $6 + 20 + 7·6c = 60$
$c = 4·5$
so UQ $= 125 + 4·5$
$= 129·5$

This would have been a hard question but occasionally asked.

Finally we could have been asked to estimate the mean

Est. Mean = $(115 \times 6 + 122·5 \times 20 + 127·5 \times 38 + 145 \times 16) \div 80$

$= 128·8$

So we have estimate median = 126·8
and estimate mean = 128·8

and the relative values of these give us the skew but this is beyond the syllabus but worth noting.

New!

26) $Q = 2^3 \times 3 \times a^3$
$R = 2^4 \times 3^2 \times a^2$
$HCF = 2^3 \times 3 \times a^2$
(we take the lower index of each factor)
so $2^3 \times 3 \times a^2 = 4056$
$a^2 = \sqrt{4056/8 \times 3}$
$a = 13$

Now for LCM we take higher index of each factor

$LCM = 2^4 \times 3^2 \times 13^2$
$= 316368$

As a check $LCM \times HCF = Q \times R$
(not a lot of people know this)
$4056 \times 316368 = 52728 \times 24336$
we we enter as
$4056 \times 316368 - 52728 \times 24336 = 0$
✓ checks

27) Expand and simplify
$(x-3)(x-4)(x+8)$

$= (x^2 - 7x + 12)(x + 8)$

$= x^3 - 7x^2 + 12x + 8x^2 - 56x + 96$
$= x^3 + x^2 - 44x + 96$

Note the layout where we line up vertically the different powers of x.