## Using Logarithms to Solve Exponential Equations

Suppose we wish to find a solution for $7^{x} = 49$ although we recognise $x = 2$
But suppose n was not an integer or even a quotient.

***Rule 1***

Take logs of both sides $\log 7^{x} = \log 49$ and the equality still holds.

***Rule 2***

Move the exponent to the front $\log 7^{x} = x \log 7$
rearranging $x = \log 49 / \log 7$ and our calculator duly gives the answer "2".

**Observations**

Logs can be to any base and ***Rule 2*** holds for any base. Calculators give two.

log means $\log_{10}$ ln means $\log_e$ where $e = 2.71828….$

The relevance of "e" need not concern us but check $x = \ln 49 / \ln 7 = 2$.

***Rule 3***

$\log_b b = 1$ Specifically $\log_{10} 10 = 1$ Check this on your calculator.

**Definition of logs**

From these three rules we can now deduce the definition of logs.

Let $y = 10^{x}$ $\qquad \log_{10} y = \log_{10} 10^{x}$

$\log_{10} y = x \log_{10} 10$ $\qquad \log_{10} y = x$ *(the definition)*

**Antilogs**

The antilog of a number is its value when you raise 10 to that number.

$\text{Antilog}_{10}(2) = 10^{2} = 100$

**Why logs Work**

We know $10^{2} \times 10^{3} = 10^{2+3} = 10^{5}$

The clue here is a multiplication has transformed into an addition.

So we take the logs of each, add and then take the "antilog".

$\log(3) + \log(2) = 0.7782..$ $\qquad \text{antilog}_{10}(0.7782\ldots) = 10^{0.7782} = 6$

And by extension an exponential say $7^{n}$ is obtained by adding n lots of log 7 and taking the antilog because $\log 7^{n} = n \log 7$

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|---|---|---|---|---|
| Let | $y$ | $= 10^x$ | $\log_{10} y$ | $= \log_{10} 10^x$ |
| | $\log_{10} y$ | $= x \log_{10} 10$ | $\log_{10} y$ | $= x$ *(the definition)* |

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