The EXP(x) function

Let us hypothesise that

1) There is a function EXP(x) that has the property such that
$^{d}/_{dx}$ { EXP (x) } = EXP (x)
A function that at every value of x the gradient is that same value.

2) That this function can be represented by a possibly infinite sum of algebraic terms.

What would such a function look like?

Seed.
We need to seed a first term. Let's assume that the term is **1**.

Now the second term is that term in **x** which when differentiated becomes **1**.

It will just be **x** because
$^{d}/_{dx}$ (x) = 1

Now consider the third term.
What when differentiated becomes x?.

It's ½ x^2 because
$^{d}/_{dx}$ (½ x^2) = x.

Now consider the fourth term.
What when differentiated becomes ½ x^2 .

It's $^{1}/_{6}$ x^3 because $^{d}/_{dx}$ ($^{1}/_{6}$ x^3) = ½ x^2

So from that initial seed of **1** we produce the algebraic series
EXP(x) = 1 + x + ½ + $^{1}/_{6}$ x^3 …

Working from the right each term when differentiated gives the term on its immediate left.

If we now differentiate the whole series the first term "disappears" and each term shuffles along one slot to the right and we end up with the same series. We can also tidy up the constants to produce

EXP (x) = 1 + x/1! + x^2/2! +x^3/3! …
We could be a pedantic and say that that first term is actually x° / 0!

Now what value does this algebraic series take if we set x = 1.

EXP (1) = 1 + 1 + 1/2! + 1/3! + 1/4! + 1/5! + 1/6! ….

It is obvious and can be rigorously proved that this series rapidly converges to a specific value.
EXP (1) = 2.718281828…
The repetition …1828 1828 is purely coincidental because we're choosing to work in number base 10. However it does mean we can remember EXP(1) to nine decimal places.

We define EXP(1) = **e**

"**e**" is a universal mathematical constant as ubiquitous as "π".
It crops up in any expression that has a connection with growth and decay.

Another Approach

Let's suppose that inflation is so high that banks are offering an interest rate on deposit of at least 100%. So if we deposit £1 in a bank a year later we will receive £2.

The equation is $(1 + {}^{1}/_{1})^{1}$

Now suppose a rival bank offers to calculate interest half yearly. So after six months your £1 is now £1.50 and that £1.50 in the next six months earns £75 so in total you receive £2.25 - that is $(1 + ½)^2$

In response your bank now offers to compound your interest **every month**. The final sum with interest becomes $(1 + {}^{1}/_{12})^{12} = 2.61$ ….

What if a bank offered to recalculate the interest every second?

1 year = 31 536 000 second and

$(1 + {}^{1}/_{31536000})^{31536000} = 2.7182816$…

Even your school calculator can push the calculation to 10^{12} and the calculator will display 2.718281828 …

This is again our universal constant **e.**

Summary

This has been the briefest of introductions. We have omitted many formal stages.

We made several major assumptions that the expression EXP(x) even existed and that the series produced actually converged to a specific value.

We also do not yet know how we can actually calculate say EXP (2) without having to work through say ten terms to get a reasonably accurate answer.

There is some tedious algebra involved but it is possible to demonstrate that

$\text{Exp}\,(x) = e^x$.

Further that
$\text{Exp}\,(x) \times \text{Exp}\,(y) = e^{x+y}$.

You can also take logs to the base e instead of base 10 and will then be able to demonstrate that

$a^x = e^{x \ln a}$ – *a key relationship.*

Now we can transform every exponential expression into a standard form.

(Yes that's what EXP stands for if you hadn't already guessed.)

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What when differentiated becomes x?.

It's $\frac{1}{2} x^2$ because
$^{d}/_{dx}$ ($\frac{1}{2} x^2$) = x.

Now consider the fourth term.
What when differentiated becomes $\frac{1}{2} x^2$.

It's $^{1}/_{6}\, x^3$ because $^{d}/_{dx}$ ($^{1}/_{6}\, x^3$) = $\frac{1}{2} x^2$

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