Quadratics - Completing the Square

Step 1

Start with $2x^2 + 4x + 2$

Graph this on your TI-83.

Divide by 2.

$x^2 + 2x + 1$

Graph this on your TI-83.

Notice both graphs have the same roots $(x + 1)^2 = 0$ so $x = -1$ a double root.

Now move $x^2 + 2x + 1$ down 4 units.

$x^2 + 2x - 3 = 0$

$(x + 1)^2 - 1 - 3 = 0$

$(x + 1)^2 = 4$

$x + 1 = \pm\sqrt{4}$

$x + 1 = \pm 2$

$x = -1 \pm 2$

$x = 1$ or $x = -3$

Step 2

Let's try $x^2 + 4x - 5 = 0$

Halve the 4x, then take off the extra term "accidentally" introduced.

$(x + 2)^2 - 4 - 5 = 0$

$(x + 2)^2 = 9$

$x + 2 = \pm 3$

$x = 1$ or $x = -5$

Step 3

Put it all together for this

$2x^2 + 8x - 10 = 0$

Take out the factor of 2.

$x^2 + 4x - 5 = 0$

Halve the x term, square and take off the extra term introduced.

$(x + 2)^2 - 4 - 5$

$(x + 2)^2 = 9$ as before

Step 4

Now try

$3x^2 + 5x - 8 = 0$

$x^2 + 5/3\,x - 8/3 = 0$

same method as before

$(x + 5/6)^2 - 25/36 - 8/3 = 0$

$(x + 5/6)^2 = 121/36$

$x + 5/6 = \pm\sqrt{(121/36)}$

$x + 5/6 = \pm 11/6$

$x = -5/6 \pm 11/6$

$x = -16/6$ or 1

$x = -8/3$ or 1

Step 5

There are other methods though.

Given $3x^2 + 5x - 8 = 0$

we might spot that $x = 1$ is a solution

$(x - 1) (ax + b) = \underline{3}x^2 + 5x - \underline{8}$

Now a must be equal to 3.

and b must be equal to 8.

Check $(x - 1) (3x + 8)$

$= 3x^2 + 5x - 8$

Step 6

Lizzie's Method

$3x^2 + 5x - 8 = 0$

$\Rightarrow 3x^2 + 5x - 24$

$\Rightarrow (x + 8) (x - 3)$

$\Rightarrow (x + 8 / 3) (x - 3 / 3)$

$\Rightarrow (3x + 8) (x - 1)$

so $x = 1$ or $x = - 8 / 3$ as before

Step 7

$ax^2 + bx + c = 0$

$ax^2 + b / a\ x + c /a = 0$

$(x^2 + b / 2a)^2 - b^2 / 4a^2 + c / a = 0$

$(ax^2 + b / 2a)^2 - b^2 / 4a^2 + c / a = 0$

$(x + b / 2a)^2 = b^2 / 4a^2 - c / a$

$(x + b / 2a)^2 = (b^2 - 4ac) / 4a^2$

$x + b / 2a = \pm \sqrt{ (b^2 - 4ac)} / 2a$

$x = \{- b \pm \sqrt{ (b^2 - 4ac)} \} / 2a$

Step 8

Cubics are a pig to solve.

Quartics are a development of quadratics

But that's it.

There are no rational solutions for quintics or higher.

A mere lad called Evariste Galois was challenged to a duel. The night before he quickly jotted down some notes covering a topic that became known as Group Theory which included this proof.

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$(x + b / 2a)^2 = b^2 / 4a^2 - c / a$

$(x + b / 2a)^2 = (b^2 - 4ac) / 4a^2$

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