

Unravelling Composite Functions

Nov2019 Paper 2 Qu19a

$$f(x) = (x + 2)^3$$

$$gf(x) = (x + 2)^{12}$$

Find $g(x)$.

How would you go about this?

There is no fixed algorithm I can give you but try this.

$$\text{That is } [(x + 2)^3]^4 = (x + 2)^{12}$$

So $g(x) = x^4$ to give

$$gf(x) = (x + 2)^{12}$$

But that's a mere warm up for

Nov2019 Paper 2 Qu19b

$$h(x) = x^2 + 5$$

$$hk(x) = 4x^2 + 5$$

find $kh(x)$

So first you have to find $k(x)$

$$[k(x)]^2 = 4x^2$$

$$\text{so } [k(x)]^2 = 4x^2$$

$$\text{So } k(x) = \sqrt{4x^2}$$

$$\text{So } k(x) = 2x$$

that is $hk(x)$

$$= (2x)^2 + 5 = 4x^2 + 5$$

Now we can calculate

$$kh(x) = 2(x^2 + 5) = 2x^2 + 10.$$

Further Examples 1

$$\text{Let } h(x) = x^2 + 7$$

$$\text{and } hk(x) = 4x^2 + 4x + 8$$

Find $k(x)$

Now assume

$$[k(x)]^2 = 4x^2 + 4x + \dots$$

$$\text{So let's try } k(x) = (2x + 1)$$

$$\begin{aligned} \text{so } hk(x) &= (2x + 1)^2 + 7 \\ &= 4x^2 + 4x + 8 \end{aligned}$$

We might then be asked find $kh(x)$

$$kh(x) = 2(x^2 + 7) = 2x^2 + 14$$

Further Example 2

$$h(x) = x^3 + 3$$

$$hk(x) = 8x^3 + 3$$

$$\text{Try } [k(x)]^3 = 8x^3 \text{ so } k(x) = 2x$$

We might be asked find $kh(x)$

$$kh(x) = 2(x^3 + 3) = 2x^3 + 6$$

Summary

There is not a specific algorithm that can be applied here. You have to make an intelligent guess and see if it works out.

Robert Goodhand

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