Logarithms

In Maths, we start with a premise and then see where it leads - no matter what.

① if $a = b^n$
if a equals b to the power n

② then by definition
$\log_b a = n$
log a to the base b equals n

Let's make b = 10 and n = x = 2
so $b^x = 10^2$ (ie 100)

Keep b = 10 but now make n = y = 3
so $b^y = 10^3$ (ie 1000)

Now $100 \times 1000 = 10^2 \times 10^3 = 10^5$
Because the base 10 is the same and both numbers are expressed as indices we find the result of **multiplying** the numbers by **adding** the indices. So we now have
Rule 1) of logarithms – the Product Rule

$\log_b (xy) = \log_b x + \log_b y$

And one should easily deduce Rule 2) the Quotient Rule as

$\log_b (x/y) = \log_b x - \log_b y$

(Log with a capital "L" has a special meaning in higher maths so for now always lower case.)

Now we are ready to use a very special result

$\log_b b = 1$
Can you see why? If not just believe.

Next we have Rule 3) – the Power Rule
$\log_b a^k = k \times \log_b a$
Can you see why? If not just believe.

With these two rules we can deduce all the rules of logs – there are actually twelve but this worksheet is just about the first three.

Back to Basics

With these last two rules we can now see where the original premise came from.

① Let $a = b^n$

Whatever a log might be just apply it to both sides but using "base b".

So $\log_b a = \log_b b^n$

Apply the Power Rule by bringing "n" to the front. This is a very powerful technique – it's a way of eliminating awkward indices in an equation.

So $\log_b a = n \times \log_b b$

Remember $\log_b b = 1$ so we have

② $\log_b a = n$

and we've re-derived what a log actually is.

rg

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