Propositional Calculus meets Monk rather than Maigret

Introduction

Propositional calculus (PC) uses letters to represent statements and then joins and manipulates those letters to form new expressions, which retain their common-sense interpretation.

But don't ever show this paper to a logician because he (or she) will still pick holes in it.

Statements

Statements are represented by (capital) letters.

A stands for (say)

"Tom committed the crime"

B stands for

"Tom went to the pub last Thursday"

~*B* stands for

"Tom **didn't** go to the pub last Thursday"

Be clear that statements can either be true or false. They start out as statements pure and simple.

Functions and Compound Sentences

Two statements can be combined together by a FUNCTION to form a compound sentence. The combination implies of itself that it's own "truth" (The Law of Identity). Whether the sentence in the final analysis is true or false depends on the truth or otherwise of the individual statements of which it is made following the rules of the connection functions.

AND

Tom is married AND has kids. (***A*** AND ***B***). Both have to be true for the compound sentence to be true. If either or both are false the sentence is false.

If he isn't married or doesn't have kids then the whole expression is false.

OR

Tom is either a thief OR a liar (***A*** OR ***B***). If either is true the compound sentence is true. Note that OR is inclusive.

Tom can be a thief AND a liar and the compound sentence is still true.

Initial Pitfalls

There are two key pitfalls in all this.

Tertium non datur

There is no third option. Either Dick went to the pub last Thursday or he didn't.

In PC (***B*** OR ***~B)*** is always true (for the logicians among you even when ***B*** is indeterminable as true or false (***B v ~B)*** is always true).

This is two-valued logic.

If the TV detective is to be believed, all murder mysteries hinge on the detective discovering that the case is an example of three-valued logic

If Dick went to the pub he didn't commit the murder. Yes he did, he slipped out the toilet window, knocked the guy over the head and slipped back before anyone missed him.

But when all said and done, all the following examples assume two-valued logic.

Misinterpreting the Implication Rule

If Tom forged the letter he'll have ink on his fingers.

If ***A*** is "forging" and ***B*** is "ink on fingers" PC writes this as

A* → *B **(if *A* then *B*)**

But ***B*** can be for other reasons (if it weren't we'd call this biconditional but don't worry about that).

Tom's got ink on his fingers so he forged the letter.

No he didn't – he's got ink on his fingers because his pen leaks.

In PC certain expressions are interchangeable

A* → *B is interchangeable with

~A OR ***B***

If Tom forged the letter he'll have ink on his fingers

is the same as

Either Tom didn't forge the letter OR he has ink on his fingers (or most importantly <u>both</u>)

Which is exactly the case here.

Tom didn't forge the letter but he's also got ink on his fingers.

PC can show the false logic of assuming Tom forged the letter just because he's got ink on his fingers.

Proofs

There are six proofs in Propositional Calculus.

In all the following exchanges be assured the desk sergeant always speaks the truth.

The rookie PC often gets it wrong. (Don't write and tell me I've assumed everyone's male – that's poetic licence)

Proof by Hypothetical Syllogism

PC Rookie

If Tom was there so was Dick

Desk Sergeant

If Dick was there so was Harry.

PC Rookie

Tom was there (assume this is true)

Desk Sergeant

Then Harry was there. Bring him in.

$$[\,(\boldsymbol{A} \rightarrow \boldsymbol{B})\ \text{AND}\ (\boldsymbol{B} \rightarrow \boldsymbol{C})\,] \rightarrow (\boldsymbol{A} \rightarrow \boldsymbol{C})$$

Proof by Disjunctive Syllogism

Desk Sergeant Either Tom or Dick committed the robbery.

PC Rookie

Tom was with me all evening.

Desk Sergeant

Dick did it. Bring him in.

$$[\,(\boldsymbol{A}\ \text{OR}\ \boldsymbol{B})\ \text{AND}\ \sim\boldsymbol{B}\,] \rightarrow \boldsymbol{A}$$

Proof By Detachment (modus ponens)

Desk Sergeant

If a jemmy was used it was forced entry

PC Rookie

A jemmy was used

Desk Sergeant

It was forced entry.

$$[\,(\boldsymbol{A} \rightarrow \boldsymbol{B})\ \text{AND}\ \boldsymbol{A}\,] \rightarrow \boldsymbol{B}$$

But watch out.

Desk Sergeant

It was forced entry.

PC Rookie

So a jemmy was used.

Desk Sergeant

No he just barged in, breaking the lock.

Proof By Indirect Reasoning (modus tollens)

Desk Sergeant

Whoever forged this letter will have ink on his fingers.

PC Rookie

Tom doesn't have ink on his fingers

Desk Sergeant

Better release him then

$[(\boldsymbol{A} \rightarrow \boldsymbol{B}) \text{ AND } \boldsymbol{\sim B}] \rightarrow \boldsymbol{\sim A}$

But as given above don't jump to the false conclusion that anyone with ink on his fingers must have forged the letter.

Proof by Contradiction (reduction ad absurdum)

This is trickier to demonstrate by a simple exchange but try this

PC Rookie

We should respect people's beliefs

Desk Sergeant

Harry is a paedophile. Do you respect his beliefs

PC Rookie

No of course not

Desk Sergeant

So you should respect people's beliefs and not respect people's beliefs. Perhaps you need to re-examine your original premise me lad.

In PC the compound sentence is

$[(\boldsymbol{A} \rightarrow \boldsymbol{B}) \text{ AND } (\boldsymbol{A} \rightarrow \boldsymbol{\sim B})] \rightarrow \boldsymbol{\sim A}$

If we assume ***A*** is true and that leads to concluding both ***B*** and **~*B*** then we must go back and examine our original assumption about ***A***.

Proof By Cases

PC Rookie

Either Tom or Dick did it that's for certain.

Desk Sergeant

Tom always works with Harry

PC Rookie

And Dick always works with Harry

Desk Sergeant

Better bring Harry in.

$\{ (\boldsymbol{A} \text{ OR } \boldsymbol{B}) \text{ AND } [(\boldsymbol{A} \rightarrow \boldsymbol{C}) \text{ AND } (\boldsymbol{B} \rightarrow \boldsymbol{C})] \} \rightarrow \boldsymbol{C}$

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