Counting Primes

Definition of a Prime

Primes are the building blocks of all numbers, the individual numbers that multiply together to produce any given number n.

eg $60 = 2 \times 2 \times 3 \times 5$

Historically "1" had been considered a prime number but no longer. The current definition is

"A prime number has exactly two divisors"

The Prime Counting Function $\pi(n)$

$\pi(n)$ represents the number of primes up to an including n.

eg $\pi(10) = 4$ (ie 2, 3, 5, 7)

The objective is to determine an expression that might approximate $\pi(n)$ without actually having to determine all the primes up to n.

Consider all numbers from 11 to 100 – how many are prime? Because we already have "2" and "5" as prime we only need consider numbers ending in 1, 3, 7 or 9.

So we list all the numbers and delete those that are not prime. We only need consider primes 2, 3, 5 and 7 as $\sqrt{100} = 10$ (why?).

11	13	17	19
~~21~~	23	~~27~~	29
31	~~33~~	37	~~39~~
41	43	47	49
~~51~~	53	~~57~~	59
61	~~63~~	67	~~69~~
71	73	~~77~~	79
81	83	~~87~~	89
~~91~~	~~93~~	97	~~99~~

The tricky one is $91 = 13 \times 7$

So $\pi(100) = 25$ –

learn and remember.

Now 40% of the numbers between 1 and 10 are prime but only 25% of the numbers between 1 and 100 are prime. This is because the higher the number the more likely it's going to be "taken out" by a bouncing prime number lower down. So the density of primes is given by a reducing probability as n increases.

The Prime Number Theorem

Around 1892, when Gauss was 15, he had a book of log tables that gave primes probably up to about 10000. So he quickly counted up to note $\pi(10) = 4$, $\pi(100) = 25$, $\pi(1000) = 168$ and $\pi(10\ 000) = 1229$.

He quickly realised the densities of primes approximated to $^{1}/_{\ln(n)}$ and presumably constructed the following table

n	$\pi(n)$	density	$^{1}/_{\ln(n)}$	error
10	4	40%	0.434	-9%
100	25	25%	0.217	+13%
1000	168	16.8%	0.145	+14%
10000	1229	12.29%	0.109	+12%

and could probably see thereafter that the percentage error kept dropping.

This can be reformulated in three ways.

Using ~ to mean "tends to"

n^{th} prime $\sim n \ln(n)$

$\pi(n) \sim {}^{n}/_{\ln(n)}$

and critically $^{\pi(n)}/_{n/\ln(n)} \rightarrow 1$ as $n \rightarrow \infty$

This is the prime number theorem.

Goodhand's Conjecture

However on reconstructing the table I noticed that

The error in the approximation of

$^{n}/_{\ln(n)}$ to $\pi(n)$

is approximately equal to $^{\pi(n)}/_{n}$

This can be seen by comparing column 3 with column 5

I therefore equated these two expressions, rearranged to a quadratic and solved to produce the more accurate elementary expression

$\pi(n) \approx \frac{1}{2} n \{ 1-\sqrt{(1 - {}^{4}/_{\ln(n)})} \}$

Despite an extensive internet search and queries on Maths websites I can find no reference to this so I claim this conjecture as mine.

Twin Primes

Examples are 11 and 13, 41 and 43

It is easy to prove the number of primes is infinite. However it is only conjectured that the number of twin primes is infinite. This remains one of the most elusive unsolved problems in number theory.

However even though both may be "infinite" there is still a difference in "magnitude".

The summation of the inverse of primes is also infinite but the summation of the inverse of twin primes is approximately 1.90216054

This is known as Brun's constant.

Both the concept and the calculation of this does defy comprehension.

(although I can sort of see how it could come about if the spacing between twins gets larger than the next decimal place)

XO *rg*

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