

An Investigation into the Fundamental Units of Nature and the Planck System of Fundamental Units

First Universal Physical Constant

1) speed light	c	velocity metres/second	$L T^{-1}$	2.9979245800E+08	m/s
----------------	---	------------------------	------------	-------------------------	-----

Fixing the speed of light as a fixed number of metres is the first step in creating a more coherent system of SI units

Properties of Free Space 4π represents the solid angle around a point

permeability of free space	μ_0		$M L Q^{-2}$	1.2566370612E-06	henrys/meter
flux density/field strength	$B = \mu H$	$4\pi \times 10^{-7}$		1.2566370614E-06	henrys/meter
permittivity of free space	ϵ_0	$\epsilon_0 = 1 / \mu_0 c^2$	$M^{-1} L^{-3} T^2 Q^2$	8.8541878192E-12	farads/meter
charge density/field strength		from $4\pi \times 10^{-7}$		8.8541878176E-12	farads/meter

The permeability of free space is no longer exactly $4\pi \times 10^{-7}$ because c has been "rounded" to a fixed 9 place figure

Other Fundamental Constants in SI Units

2) gravitational constant	G	Force = $G m_1 m_2 / r^2$	$M^{-1} L^3 T^{-2}$	6.6743000000E-11	Newton-metre ² /kg ²
Planck constant	h	$E = h c / \lambda$	$M L^2 T^{-1}$	6.6260701500E-34	joule-sec
3) reduced Planck constant	$\hbar$	unit of action	$M L^2 T^{-1}$	1.0545718176E-34	joule-sec
4) Boltzmann constant	k_B	$p V = N_A k_B T$	$M L^2 T^{-2} \Theta^{-1}$	1.3806490000E-23	joules/degree Kelvin
5) Coulomb constant	k_e	$F = k_e q_1 q_2 / r^2$	$M L^3 T^{-2} Q^{-2}$	8.9875517923E+09	newton-metre ² /coulomb ²

Including the Coulomb constant extends the system into electromagnetism by introducing ϵ_0 and hence k_e

For completeness there are two other fundamental constants of nature now given exact values

6) Avogadro's constant	N_A	The number of particles in one mole		6.0221407600E+23	dimensionless
7) Elementary charge	e	$e^2 = 2 h \alpha / \mu_0 c$		1.6021766340E-19	coulomb

This relationship therefore fixes the fine structure constant α

Fine structure constant	α^{-1}	$\alpha^{-1} = 2 h / \mu_0 c e^2$	fixed by μ_0	1.3703599918E+02	dimensionless
-------------------------	---------------	-----------------------------------	------------------	------------------	---------------

Planck Fundamental Constants

Planck Mass	m_P	$\cong 32 \mu\text{grams}$	$\sqrt{(\hbar c / G)}$	2.1764343427E-08	kilograms
Planck Length	l_P	quantum limit	$\sqrt{(\hbar G / c^3)}$	1.6162550244E-35	metres
Planck Time	t_P	quantum limit	$\sqrt{(\hbar G / c^5)}$	5.3912464483E-44	seconds
Speed Light (Planck speed)	c	maximum limit	l_P / t_P	2.9979245800E+08	metres/second
Planck Temperature	T_P	quantum limit	$\sqrt{(\hbar c^5 / G k_B^2)}$	1.4167841622E+32	kelvin
Planck Charge	q_P		$\sqrt{(\hbar c / k_e)}$	1.8755460378E-18	coloumb

Because μ_0 is no longer exactly $4\pi \times 10^{-7}$ traditional rearrangements give slightly different values. That's the price of fixing c.

Recalculating Fundamental Constants from Dimensions in Planck units - recreates original values

gravitational constant	G	$F = G m_1 m_2 / r^2$	$M^{-1} L^3 T^{-2}$	6.6743000000E-11	newton-metre ² /kg ²
gravitational constant	G	$l_P^3 c^2 / \hbar t_P$	$M^{-1} L^3 T^{-2}$	6.6743000000E-11	newton-metre ² /kg ²
reduced Planck constant	$\hbar$		$M L^2 T^{-1}$	1.0545718176E-34	joule-sec
Boltzmann constant	k_B	$k_e = k_B \times T$	$M L^2 T^{-2} T^{-1}$	1.3806490000E-23	joules / degree
Coulomb constant	k_e	$1 / 4 \pi \epsilon_0$	$M L^3 T^{-2} Q^{-2}$	8.9875517923E+09	newton-metre ² /coulomb ²

Planck Derived Constants

			Calculated	Derived from Fundamental	
Planck Area	l_P^2	$\hbar G / c^3$	2.6122803040E-70	2.6122803040E-70	metres ²
Planck Volume	l_P^3	$\sqrt{(\hbar^3 G^3 / c^9)}$	4.2221111665E-105	4.2221111665E-105	metres ³
Planck Momentum	$m_P c$	$\sqrt{(\hbar^3 c^3 / G)}$	6.5247860128E+00	6.5247860128E+00	kg-metre/sec
Planck Acceleration a_P	c / t_P	$\sqrt{(c^7 / \hbar G)}$	5.5607262787E+51	5.5607262787E+51	metres/sec ²
Planck Force $\mathcal{F}_P$	$m_P \times a_P$	c^4 / G	1.2102555643E+44	1.2102555643E+44	newtons
Planck Energy $\mathcal{E}_P$	$\mathcal{F}_P \times l_P$	$\sqrt{(\hbar c^5 / G)}$	1.9560816367E+09	1.9560816367E+09	joules
Planck Density - ρ_P	m_P / l_P^3	$c^5 / \hbar G$	3.5176729424E-36	5.1548485032E+96	kg / metre ³
Planck Frequency f_P	$1 / t_P$	$\sqrt{(c^5 / \hbar G)}$	1.8548586298E+43	1.8548586298E+43	per second

Re-deriving Planck Units			Calculated	Given	Error
Planck Mass m_P	ℓ_P	$\hbar / \ell_P c$	2.1764343427E-08	2.1764343427E-08	
Planck Mass m_P	t_P	$\hbar / t_P c^2$	2.1764343427E-08	2.1764343427E-08	0.000E+00
Planck Mass m_P	T_P	$T_P k_B / c^2$	2.1764343427E-08	2.1764343427E-08	-3.040E-16
Planck Mass m_P	q_P	$q_P \times \sqrt{(k_e/G)}$	2.1764343427E-08	2.1764343427E-08	0.000E+00
Planck Length ℓ_P	m_P	$\hbar / m_P c$	1.6162550244E-35	1.6162550244E-35	0.000E+00
Planck Length ℓ_P	t_P	$c t_P$	1.6162550244E-35	1.6162550244E-35	1.654E-16
Planck Length ℓ_P	T_P	$\hbar c / T_P k_B$	1.6162550244E-35	1.6162550244E-35	0.000E+00
Planck Length ℓ_P	q_P	$\hbar / q_P c \times \sqrt{(G/k_e)}$	1.6162550244E-35	1.6162550244E-35	0.000E+00
Planck Time t_P	m_P	$\hbar / m_P c^2$	5.3912464483E-44	5.3912464483E-44	-1.847E-16
Planck Time t_P	ℓ_P	ℓ_P / c	5.3912464483E-44	5.3912464483E-44	-1.847E-16
Planck Time t_P	T_P	$\hbar / T_P k_B$	5.3912464483E-44	5.3912464483E-44	0.000E+00
Planck Time t_P	q_P	$\hbar / q_P c^2 \times \sqrt{(G/k_e)}$	5.3912464483E-44	5.3912464483E-44	-3.694E-16
Planck Temperature T_P	ℓ_P	$\hbar c / \ell_P k_B$	1.4167841622E+32	1.4167841622E+32	1.271E-16
Planck Temperature T_P	m_P	$m_P c^2 / k_B$	1.4167841622E+32	1.4167841622E+32	0.000E+00
Planck Temperature T_P	t_P	$\hbar / t_P k_B$	1.4167841622E+32	1.4167841622E+32	0.000E+00
Planck Temperature T_P	q_P	$q_P c^2 / k_B \times \sqrt{(k_e/G)}$	1.4167841622E+32	1.4167841622E+32	1.271E-16
Planck Charge q_P	m_P	$m_P \times \sqrt{(G/k_e)}$	1.8755460378E-18	1.8755460378E-18	0.000E+00
Planck Charge q_P	ℓ_P	$\hbar / \ell_P c \times \sqrt{(G/k_e)}$	1.8755460378E-18	1.8755460378E-18	-2.054E-16
Planck Charge q_P	t_P	$\hbar / t_P c^2 \times \sqrt{(G/k_e)}$	1.8755460378E-18	1.8755460378E-18	0.000E+00
Planck Charge q_P	T_P	$T_P k_B / c^2 \times \sqrt{(G/k_e)}$	1.8755460378E-18	1.8755460378E-18	-4.107E-16
Planck Charge q_P	q_P	$\sqrt{(4 \pi \epsilon_0 \hbar c)}$	1.8755460385E-18	1.8755460378E-18	3.635E-10
Planck Charge q_P	q_P	$\sqrt{(4 \pi \hbar / \mu_0 c)}$	1.8755460385E-18	1.8755460378E-18	3.635E-10
Planck Charge q_P	q_P	$e / \sqrt{\alpha}$	1.8755460385E-18	1.8755460378E-18	3.635E-10
Planck Force F_P	m_P / ℓ_P	$G m_P^2 / \ell_P^2$	1.2102555643E+44	1.2102555643E+44	1.637E-16
Planck Force F_P	m_P / t_P	$m_P c / t_P$	1.2102555643E+44	1.2102555643E+44	0.000E+00
Planck Force F_P	$\ell_P t_P$	$G \hbar^2 / \ell_P^2 t_P^2 c^4$	1.2102555643E+44		
Planck Force F_P	$\ell_P t_P$	$\hbar / \ell_P t_P$	1.2102555643E+44	1.2102555643E+44	0.000E+00
Planck Force F_P	ℓ_P	$G \hbar^2 / \ell_P^4 c^2$	1.2102555643E+44		
Planck Force F_P	F_P	$(4\pi\epsilon_0)^{-1} \times (q_P / t_P)^2$	1.2102555635E+44	1.2102555643E+44	-7.269E-10
Planck Force F_P	F_P	E_P / ℓ_P	1.2102555643E+44	1.2102555643E+44	0.000E+00
Planck Energy E_P	E_P	$\hbar / t_P$	1.9560816367E+09	1.9560816367E+09	0.000E+00
Planck Energy E_P	E_P	$m_P \ell_P^2 / t_P^2$	1.9560816367E+09	1.9560816367E+09	-1.219E-16
Planck Energy E_P	E_P	$k_B T_P$	1.9560816367E+09	1.9560816367E+09	-2.438E-16
Planck Energy E_P	E_P	$F_P \ell_P$	1.9560816367E+09	1.9560816367E+09	0.000E+00
Planck Energy E_P	E_P	$m_P c^2$	1.9560816367E+09	1.9560816367E+09	0.000E+00
Planck Density	ρ_P	$\hbar t_P / \ell_P^5$	5.1548485032E+96	5.1548485032E+96	3.680E-16
Interpretation of Planck Units					
Planck Mass	m_P	upper limit on the mass elementary particle / lower limit on the mass of a black hole			
Planck Length	ℓ_P	gravitational limit of quantum theory / quantum limit of general relativity			
Planck Time	t_P	gravitational limit of quantum theory			
Planck Temperature	T_P	upper limit of temperature (absolute hotness)			
Planck Density	ρ_P	limiting density of matter			
Volume universe when compressed	1.93992E-37		Volume hydrogen atom		6.2E-31
so you could fit 3 million universes inside a hydrogen atom which is pretty amazing by any standards.					$\propto$ rg