

Paper 3 June 2020

1) The symbol $\cup$ stands for UNION
 $\cap$ stands for INTERSECTION
 colloquially $\cup$ roughly means OR
 and $\cap$ roughly means AND.
 so $A \cup B$ means A union B
 which "translates" as
 A or B or both

2) $y = mx + c$
 $\hookrightarrow$ this is the gradient
 so $y = \frac{1}{2}x + 3$
 $\hookrightarrow$ gradient = $\frac{1}{2}$
 so a parallel line is
 $y = \frac{1}{2}x$

3) $\frac{320}{80} = 4$? what does this mean.
 lets try something simple
 $\frac{20}{100} = 0.2 \Rightarrow 20\%$
 so we just multiply by 100.
 $\frac{320}{80} = 400\%$

4) $P(4 \text{ heads})$ means
 a head then another head etc.
 The word we think of is AND.
 In probability AND means $\times$
 OR means $+$
 (it's back to front from what you might expect)
 $P(4 \text{ heads}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
 $= \frac{1}{16}$

5) 18000 to nearest 1000
 $17500 < n < 18500$
 Minimum attendance
 17500
 Maximum attendance
 18500

6. This is a construction
 to bisect the angle ABC

7) $x^2 = 32^2 + 60^2$
 $x = 68$

This is actually a 8/15/17
 triangle in disguise.

8) Distance = speed $\times$ time
 $= 12 \text{ mph} \times \frac{1}{2} \text{ hour}$
 So library is 6 miles away.
 Speed home = $\frac{\text{distance}}{\text{time}}$
 $= \frac{6}{3/4}$
 $= 8 \text{ mph}$

we could have converted to minutes
 then back to hours but these values
 are so simple it's easier to work
 through in fractions of an hour.

9) Large Triangle Small Triangle.
 $20 : 16$
 $\times 6 \left(\begin{array}{l} 2 : 1.6 \\ 12 : 9.6 \end{array} \right) \times 6$
 $a = 9.6 \text{ cm}$

10) $4(2c + 3) - (5c - 1)$
 $= 8c + 12 - 5c + 1$
 watch out two negatives
 make a positive
 $\Rightarrow 3c + 13$

11)

red	blue	green	total
0.18	0.62	0.20	1.00

 freq green = spin $\times$ relative frequency
 $= 350 \times 0.2$
 $= 70$

12)

Bedrooms	Houses
1	7
2	a
3	b
4	c
5	8

Median number bedrooms 3.5
Median house is 13/14

$$\text{so } a+b = 13-7$$

$$\text{so } a+b = 6$$

$$\text{so try } a=b=3 \text{ and } c=5$$

Bedrooms	Houses	
1	7	
2	3	} 13
3	3	
4	5	} 13
5	8	
	26	

That seems to work fine

13a) Simplify $\frac{25a}{8} \times \frac{2a}{5}$

$$= \frac{50a^2}{8 \times 5}$$

$$= \frac{10a^2}{8} = \frac{5a^2}{4}$$

13b) Simplify $\frac{6c+10}{2}$

Sofia says divide 6c by 2 then subtract 10.

This is wrong. She has to divide 6c and 10 by 2 to get $3c+5$

14) She is incorrect. You cannot just add percentages

$$60 \times 0.85 = 51$$

$$40 \times 0.9 = 36$$

$$\text{New perimeter} = 174$$

$$\text{Old perimeter} = 200$$

$$\frac{174}{200} = 0.87 \text{ 13\% reduction.}$$

She'd have been better to average although that would still be incorrect

15) Treat inequalities w/ equalities remembering

if we flip left and right flip inequality
if we multiply both sides by -1 flip w/ word.

$$4 > 11 - \frac{x}{3}$$

$$\frac{x}{3} + 4 > 11$$

$$\frac{x}{3} > 7$$

$$x > 21$$

16)

Goals	Freq.	Mid	Product
0-4	6	2	12
5-9	11	7	77
10-14	3	12	36
			125

$$\text{Estimated mean} = \frac{125}{20} = 6.25$$

17)

$$\begin{array}{ll} \text{Shaded} & \text{Large} \\ (12-x)^2 & 12(x+2) \end{array}$$

$$4 \times (12-x)^2 = 12(x+2)$$

$$96 - 8x = 12x + 24$$

$$20x = 72$$

$$x = 3.6$$

$$\text{check } 12 \times 5.6 = 67.2$$

$$8 \cdot 4 \times 2 = 16.8$$

$$16.8 \times 4 = 67.2 \checkmark \text{ checks}$$

18)

$$\text{Pressure} = 30 \text{ pounds/inch}^2$$

$$30 \frac{\text{lb}}{\text{in}^2} \times \frac{0.45 \text{ kg}}{11 \text{ lb}} \times \frac{1 \text{ in}^2}{2.54^2 \text{ cm}^2}$$

(note lb is the symbol for pounds)

$$= \frac{30 \times 0.45}{2.54^2} \text{ kg/cm}^2$$

$$= 2.09 \text{ kg/cm}^2$$

13) Description of shaded area

$$x < 1 \quad y > -3$$

20) Draw boxplot

This should be straightforward.

$$IQR \text{ Amin} = 6$$

$$IQR \text{ Ben} = 3$$

Ben has the more consistent score.

21a) PAB is an isosceles triangle

$$\angle APB = 180 - 2 \times 71$$

$$= 38^\circ$$

b) $\angle CXD = 360 - 204$
 $= 156^\circ$

if X is centre of circle

$$\angle CXD = 2 \times 78^\circ$$

$$= 156^\circ$$

so X is the centre circle.

22) children adults total

	children	adults	total
X	120	130	250
Y	17n	15n	32n

$$P(2 \text{ children}) = \frac{120}{250} \times \frac{17n}{32n}$$

$$= 0.255$$

$$P(2 \text{ adults}) = \frac{130}{250} \times \frac{15n}{32n}$$

$$= 0.24375$$

In fact I could demonstrate this directly because

$$120 \times 17 > 130 \times 15$$

Picking two children is more likely.

23) $\vec{JK} = \vec{JL} - \vec{LK}$
 $= 10b - 7a$
 $\vec{JM} = 5b - 3\frac{1}{2}a$

24) Rate change = $\frac{\Delta y}{\Delta x}$

Draw tangent and measure

$$\text{Rate of change} = \frac{11 - 4.4}{5 - 0}$$

$$= 1.32$$

Your calculation may differ slightly. The examiner tends to be generous but draw carefully.

b) The average rate of change between A and B is negative.

25) $x^2 + y^2 = 9$ so radius = 3
diameter = 6.

26) let $l = 3.47$
so $10l = 34.777 \dots$

$$\begin{array}{r} 34.777 \\ - 3.477 \\ \hline 31.300 \end{array}$$

so $9l = 31.3$

$$l = 313/90$$

which is readily confirmed on your calculator.

27) $y = (x-1)^2 - 6$

The turning point is (1, -6)
if you don't know why then ask for the explanation sheet.

28) $y = 4x - 1$
gradient perpendicular is $-\frac{1}{4}$
Now we $y - y_1 = m(x - x_1)$
 $y - 5 = -\frac{1}{4}(x - 8)$
 $y - 5 = -\frac{1}{4}x + 2$
 $y = -\frac{1}{4}x + 7$

29) The shape is a semicircle plus a triangle.

Let's first find AC.

For this we use the cosine rule.

$$b^2 = a^2 + c^2 - 2ac \cos B.$$

The cosine rule is an extension of Pythagoras's theorem for non-right angled triangles.

When $B = 90^\circ$ $\cos B = 0^\circ$

and we just have $b^2 = a^2 + c^2$

When $B < 90$ we take a bit off because $\cos B$ is positive

When $B > 90$ we add a bit on because $\cos B$ is negative,

$$\text{Here } b^2 = 8^2 + 10^2 - 2 \cdot 8 \cdot 10 \cos 114^\circ$$

$$= 164 - 160 \cos 114^\circ$$

$$b = 15.1$$

$$\text{so radius} = \frac{15.1}{2}$$

and semicircle area is

$$\frac{1}{2} \left(\frac{15.1}{2} \right)^2 \pi$$

$$= 90.0 \text{ cm}^2$$

Area triangle we use $\frac{1}{2} ab \sin C$ rule.

$$= \frac{1}{2} 8 \times 10 \times \sin 114^\circ$$

$$= 36.5 \text{ cm}^2$$

$$\text{Total area} = 126.5 \text{ cm}^2$$

Can we do this in one calculation?

$$8^2 + 10^2 - 2 \times 8 \times 10 \times \cos 114^\circ =$$

$$\sqrt{\text{ANS}} =$$

$$\text{ANS} \div 2 =$$

$$\text{ANS}^2 \div 2 \times \pi =$$

$$\text{ANS} + 40 \sin 114^\circ =$$

$$126.5 \text{ cm}^2$$

It is possible with careful entry but risky.

$$f(x) = \frac{1}{2}x$$

$$g(x) = x - x^2$$

$$f^{-1}(x) = 2x$$

$$gf(x) = \frac{1}{2}x - \left(\frac{1}{2}x\right)^2$$

$$= \frac{1}{2}x - \frac{1}{4}x^2$$

$$\text{so if } f^{-1}(x) = gf(x)$$

$$2x = \frac{1}{2}x - \frac{1}{4}x^2$$

$$8x = 2x - x^2$$

$$x^2 + 6x = 0$$

$$x(x+6) = 0$$

$$\text{so we have } x = 0$$

$$\text{or } x = -6$$

It at the stage $x^2 = -6x$ you just divide through by x there is the condition that $x \neq 0$ which of course it can. so select to gather all the terms together and then factorise.