Second Moment - Variance

We have studied the expression $\Sigma (x_i - \mu) / n$

This is called the first moment

We have demonstrated that this summation is zero.

This is the definition of the mean μ

We actually calculate μ from the expression $\Sigma (x_i) / n$

For the second moment we study the expression

$$\Sigma (x_i - \mu)^2 / n$$

Now replace μ with any arbitrary value k

$$= \Sigma (x_i - k)^2 / n$$

$$= \Sigma (x_i^2 + k^2 - 2kx_i) / n$$

$$= (\Sigma x_i^2 + \Sigma k^2 - \Sigma 2kx_i) / n$$

$$= (\Sigma x_i^2 + nk^2 - 2k\Sigma x_i) / n$$

Dividing through by n and rearranging

$$= k^2 - 2k\Sigma x_i / n + \Sigma x_i^2 / n$$

Now split this expression into two and introduce the term $(\Sigma x_i / n)^2$

which is actually μ^2

$= k^2 - 2k\Sigma x_i / n + (\Sigma x_i / n)^2 + \Sigma x_i^2 / n - (\Sigma x_i / n)^2$

$= (k - \Sigma x_i / n)^2 + \Sigma x_i^2 / n - (\Sigma x_i / n)^2$

Now the minimum value of this expression occurs when

$$k = \Sigma x_i / n$$

So the minimum value of the second moment occurs when $k = \mu$

and its value is $\Sigma x_i^2 / n - (\Sigma x_i / n)^2$

ie the mean of the squares minus the square of the mean

rg

Second Moment - Variance

We have studied the expression $\Sigma (x_i - \mu) / n$

This is called the first moment

We have demonstrated that this summation is zero.

This is the definition of the mean μ

We actually calculate μ from the expression $\Sigma (x_i) / n$

For the second moment we study the expression

$$\Sigma (x_i - \mu)^2 / n$$

Now replace μ with any arbitrary value k

$$= \Sigma (x_i - k)^2 / n$$

$$= \Sigma (x_i^2 + k^2 - 2kx_i) / n$$

$$= (\Sigma x_i^2 + \Sigma k^2 - \Sigma 2kx_i) / n$$

$$= (\Sigma x_i^2 + nk^2 - 2k\Sigma x_i) / n$$

Dividing through by n and rearranging

$$= k^2 - 2k\Sigma x_i / n + \Sigma x_i^2 / n$$

Now split this expression into two and introduce the term $(\Sigma x_i / n)^2$

which is actually μ^2

$= k^2 - 2k\Sigma x_i / n + (\Sigma x_i / n)^2 + \Sigma x_i^2 / n - (\Sigma x_i / n)^2$

$= (k - \Sigma x_i / n)^2 + \Sigma x_i^2 / n - (\Sigma x_i / n)^2$

Now the minimum value of this expression occurs when

$$k = \Sigma x_i / n$$

So the minimum value of the second moment occurs when $k = \mu$

and its value is $\Sigma x_i^2 / n - (\Sigma x_i / n)^2$

ie the mean of the squares minus the square of the mean

rg