Conditional Probability "Venn" v "Tree" v Algebra Iss2

Edexcel Paper 2 Sample Question 14

X and Y are two events such that

$$P(X \text{ and } Y) = 0.2 \quad P(X|Y) = 0.4 \quad P(Y|X) = 0.5$$

Hiki says "X and Y are independent events". Do you agree? Explain why?

2 marks

Preliminaries

Be warned just because we're given a conditional probability, do not deduce there is any "association" between X and Y and certainly do not assume any causal connection.

I think the question would have been more challenging to have Hiki say "X and Y are **dependent** events" because that is the natural assumption. But maybe the question is already tricky enough.

The Edexcel Revision Book gives two key equations.

Page 89 P (A **AND** B) = P(A) × P(B)

applies when A and B are independent.

Page 92 P (A **AND** B) = P(A) × P(B|A gives the general form.

But to demonstrate (P(B|A) = P(B) is **sufficient** to indicate A and B are independent.

To me that seems an intuitive conclusion to reach anyway.

How To Approach

It is tempting to draw a Venn diagram. Be warned - Venn diagrams on the face of it give no indication as to whether A and B are independent or dependent. Avoid in these questions.

Should we draw a tree diagram? There are three disadvantages.

1) We have to interpret the wording to decide which branch for which probabilities.

2) While we can easily enter P(X|Y) by drawing the Y branch first, we then can't immediately determine P(Y|X). Switching around won't help.

3) To demonstrate independence ultimately we have to enter probabilities for all six branches of the tree diagram. For just two marks for this question, that's a lot of work and time taken.

Tree diagrams are good for reading off lots of quick answers. This isn't one of them.

The Algebraic Approach

Write P(X **AND** Y) = P(X|Y) × P(Y) and enter what you know.

It doesn't actually matter which way round you write X and Y.

> 0.2 = 0.4 × P(Y) which gives P(Y) = 0.5.
>
> But now note that we've already been given that P(Y|X) = 0.5

So knowledge of X doesn't affect the probability of Y and therefore that is sufficient to demonstrate that X and Y are independent. To be safe, spell it out to the examiner.

For completeness we could write P(Y **AND** X) = P(Y|X) × P(X) now to deduce
P(X) = 0.4 , which incidentally is now equal to P(X|Y) = 0.4, mirroring the result above.

Now check P(X) × P(Y) = 0.4 × 0.5 = 0.2 which is the given result so independence is proved. But I do not think this extra step is necessary to obtain the full marks.

rg

Conditional Probability "Venn" v "Tree" v Algebra Iss2

Edexcel Paper 2 Sample Question 14

X and Y are two events such that

$$P(X \text{ and } Y) = 0.2 \quad P(X|Y) = 0.4 \quad P(Y|X) = 0.5$$

Hiki says "X and Y are independent events". Do you agree? Explain why?

2 marks

Preliminaries

Be warned just because we're given a conditional probability, do not deduce there is any "association" between X and Y and certainly do not assume any causal connection.

I think the question would have been more challenging to have Hiki say "X and Y are **dependent** events" because that is the natural assumption. But maybe the question is already tricky enough.

The Edexcel Revision Book gives two key equations.

Page 89 P (A **AND** B) = P(A) × P(B)

applies when A and B are independent.

Page 92 P (A **AND** B) = P(A) × P(B|A gives the general form.

But to demonstrate (P(B|A) = P(B) is **sufficient** to indicate A and B are independent.

To me that seems an intuitive conclusion to reach anyway.

How To Approach

It is tempting to draw a Venn diagram. Be warned - Venn diagrams on the face of it give no indication as to whether A and B are independent or dependent. Avoid in these questions.

Should we draw a tree diagram? There are three disadvantages.

1) We have to interpret the wording to decide which branch for which probabilities.

2) While we can easily enter P(X|Y) by drawing the Y branch first, we then can't immediately determine P(Y|X). Switching around won't help.

3) To demonstrate independence ultimately we have to enter probabilities for

all six branches of the tree diagram. For just two marks for this question, that's a lot of work and time taken.

Tree diagrams are good for reading off lots of quick answers. This isn't one of them.

The Algebraic Approach

Write P(X **AND** Y) = P(X|Y) × P(Y) and enter what you know.

It doesn't actually matter which way round you write X and Y.

> 0.2 = 0.4 × P(Y) which gives P(Y) = 0.5.
>
> But now note that we've already been given that P(Y|X) = 0.5

So knowledge of X doesn't affect the probability of Y and therefore that is sufficient to demonstrate that X and Y are independent. To be safe, spell it out to the examiner.

For completeness we could write P(Y **AND** X) = P(Y|X) × P(X) now to deduce
P(X) = 0.4 , which incidentally is now equal to P(X|Y) = 0.4, mirroring the result above.

Now check P(X) × P(Y) = 0.4 × 0.5 = 0.2 which is the given result so independence is proved. But I do not think this extra step is necessary to obtain the full marks.

xo *rg*