

Perpendicular Bisector

Finding the general solution to the perpendicular bisector is a worthwhile exercise because it pulls together all the coordinate geometry strands.

Let the given points be (x_1, y_1) and (x_2, y_2)

So the midpoint of the line joining is $[\frac{1}{2}(x_1 + x_2), \frac{1}{2}(y_1 + y_2)]$

And the gradient of that line is $(y_2 - y_1) / (x_2 - x_1)$

So the gradient of the perpendicular is $(x_1 - x_2) / (y_2 - y_1)$ [$m_1 m_2 = -1$]

So using the equation $(y - y_1) = m(x - x_1)$

we get $y - \frac{1}{2}(y_1 + y_2) = [(x_1 - x_2) / (y_2 - y_1)] [x - \frac{1}{2}(x_1 + x_2)]$

multiplying through by $2(y_2 - y_1)$ we get

$$2y(y_2 - y_1) - (y_2 + y_1)(y_2 - y_1) = 2x(x_1 - x_2) - (x_1 + x_2)(x_1 - x_2)$$

And collecting and rearranging and taking care with relevant signs we produce

$$2y(y_2 - y_1) + 2x(x_2 - x_1) = (y_2^2 - y_1^2) + (x_2^2 - x_1^2)$$

or

$$y(y_2 - y_1) + x(x_2 - x_1) = \frac{1}{2} \{ (y_2^2 - y_1^2) + (x_2^2 - x_1^2) \}$$

which looks pretty neat by any standards.

✪ rg

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