Paper 3 Nov 18

1) $\begin{bmatrix} 0 \\ 4 \end{bmatrix}$ moves up.

This is a pure convention in the way we present and arrange the x-y coordinates

2)

$$\frac{1.75\ km}{700\ m} \times \frac{1000\ m}{1\ km} = \frac{5}{2}$$

3)

0	1	2	3	4	5
-5	3	11	19	27	

-8 8 8 8

this is jumping number.

This is what we got "for free"

so $y = 8n - 5$

jumping

free

4) LCM (20, 30, 40)

As 40 is a multiple of 20 we just investigate 30 and 40

30	40
60	80
90	120
120	

LCM.

LCM (20, 30, 40) = 120

5)

$110.5 \leq 110cm < 109.5$

check $\frac{109.5 + 110.5}{2} = 110.$

6) Plot and read off.

Despite the examiner elsewhere insisting we should never extrapolate, here we are being asked to do exactly that.

The examiner wants an answer 4300 or thereabouts.

7) $k = n^2 + 9n + 1$

start from 9

9 => 163
8 => 137
7 => 113
6 => 91
5 => 71
4 => 53
3 => 37
2 => 23
1 => 11

$91 = 7 \times 13$

In compiling primes under 100 - there are 25 - 91 is often mistakenly classed as prime - the factors overlooked. This is a tedious question. Newer calculators that are allowed in examinations will actually factorise numbers so why is this question in Paper 3.

8)

what is £600 reduces 20% over 5 months

$600 \times (0.8)^5 = £196$

He'll still owe £196. Theoretically by this method he'll never pay off the debt. He should have said "Every month I'll pay off £120"

9) This is an extremely interesting question. It is a provable fact that the wiggles neither increase or decrease the total distance - they simply equate to a semicircle.

So 1 lap $= 1.5 + 0.9\pi$

and $\frac{305}{1.5 + 0.9\pi} = 70.5$

so 71 laps.

10) solve $8 > 3 - \frac{1}{2}x$

Treat inequalities as equality with extra proviso.

a) if you flip them flip sign

b) if you $\times -1$ flip sign.

$\frac{1}{2}x > 3 - 8$

$x > -10$

11) cosine $x = 9/10$

$x = \cos^{-1} 0.9$

$= 25.8°$

12) Lewis should have attempted to draw a smooth curve.

13) $E(\text{tails}) = (1-0.6) \times 500$

$= 200$

14) Mean mass 19 players = 82kg.

New mean mass

$= \frac{82 \times 19 + 93}{20}$

$= 82.55.$

Hint enter $82 \times 19 + 93$ =

ANS $\div 20$ =

15)

Lampshade A

Area $= 2\pi\, 8 \times 22$

$= 0.1105\ m^2$

Lampshade B

$= 4 \times \frac{1}{2} \times 15 \times 24$

$= 0.072\ m^2$

Cost A = ANS × 400 + £3.50

= £47.23

Cost B = ANS × 400 + £7.50

= £36.30

Cost A : B

47.23 : 36.30

$\div 36.3$ 1.31 : 1

16) Running Club

P(blue eyes/female) = 0.38

P(blue eyes/male) = 0.60

blue eyed females = 50×0.38

blue eyed males = 80×0.60

blue eyed = 19 + 48

= 67

P(blue eyed) $= \frac{67}{130} = 0.52$

$0.52 > 0.5$

The examiner gives the hint as a check that we have performed the correct calculation. They are making life easy for us.

17) $w = 3/5\sqrt{x}$

$w^2 = \frac{3 \times 3}{5 \times 5 \times x} = \frac{9}{25x}$

which is a given option.

18)

Age	Frequency	Width	Density
$10 \le x < 15$	8	5	1.6
$15 \le x < 25$	24	10	2.4
$25 \le x < 40$	30	15	2.0
$40 \le x < 70$	39	30	1.3

Draw histogram.

Note the frequency of each bar is given by the area.

i.e band width × freq density

As an extra task to demonstrate we fully understand histograms locate the lower quartile

median

and upper quartile

19) Maximum length ribbon left on roll

$= 30.25 - 5.75$

$= 24.5$ metres

20) $y = 2(x-1)^2 - 5$

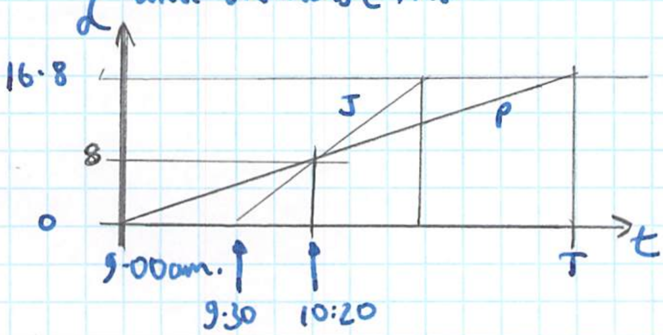
the $(x-1)$ term moves the function 1 unit right.

so $(x+1)$ term moves the function 1 unit left.

If you have a graphical calculator enter these two functions and observe exactly what is happening

$$f(x) = 2(x+1)^2 - 5$$
$$= 2(x^2+2x+1) - 5$$
$$= 2x^2 + 4x + 2 - 5$$
$$= 2x^2 + 4x - 3$$

21) It is best to sketch a graph and visualise this



Time T $= \frac{16\cdot8}{6} = 2\cdot8 + 9{\cdot}00$am.

but do we need this?

They intersect at $t = 1\frac{1}{3}$

so $d = \frac{4}{3} \times 6 = 8$

so for Joe's line we have 2 points $(\frac{1}{2}, 0)$ and $(1\frac{1}{3}, 8)$

so speed $= \frac{\Delta d}{\Delta t} = \frac{8}{5/6}$

$= 48/5$ k/hr.

time to complete $= \frac{16\cdot8}{48} \times 5$

$= 1\,3/4$ hrs

so. Joe finishes at $9{:}30 + 1\,3/4$

$= 11{:}15$ am

22) $x_{n+1} = \frac{(x_n)^3 - 2}{10}$

$x_1 = -1$

$x_2 = \frac{(-1)^3 - 2}{10}$

$= \frac{-3}{10}$

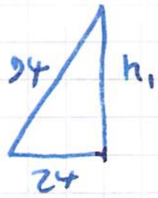
$x^2 = \frac{\left(\frac{-3}{10}\right)^3 - 2}{10}$

$= -0{\cdot}2027$

and we continue the iteration.

solution is $-0{\cdot}20081$

23)



$h_1^2 = 94^2 - 24^2$

$h_1 = 90{\cdot}88$

by similar triangles

$h = 1\frac{1}{2}h_1$

$= 136$cm.

24) sphere volume $= \frac{4}{3}\pi(2x)^3$

$= \frac{32}{3}\pi x^3$

cone volume $= \frac{1}{3}\pi(3x)^2 h$

$= 3\pi x^2 h$

a) volumes are equal

$32x = 9h$

$3x = \frac{27}{32}h$

$3x : h$

$\frac{27}{32} : 1$

$27 : 32$

25) If the quadrilateral is reflected in the vertical line $x = 4$ then the invariant points are B and C

26) $f(x) = \frac{2x+3}{x-4}$

$y = \frac{2x+3}{x-4}$

The task is to gather all the x terms on one side, factorise out the x and move that factor back to the right.

$y(x-4) = 2x+3$

$xy - 4y = 2x+3$

$xy - 2x = 4y+3$

$x(y-2) = 4y+3$

$x = \frac{4y+3}{y-2}$

so $f^{-1}(x) : \frac{4x+3}{x-2}$

Not too difficult if you know where you're heading from the outset.

27) ① $y = 3x + p$ ①

② $x^2 + y^2 = 53$ ②

and they intersect at A B

Now $y^2 = 53 - x^2$ ②

$y^2 = (3x+p)^2$ ①

$y^2 = 9x^2 + 6px + p^2$ ①

Equate ① and ②

$9x^2 + 6px + p^2 = 53 - x^2$

$10x^2 + 6px + p^2 - 53 = 0$

which is the equation we were asked to find. So far so good.

b) The coordinates of A are (2, 7) Find coordinates of B

First step is to plug these coordinates into ① - the simpler equation - to find p

$7 = 3 \times 2 + p$

so $p = 1$

so $10x^2 + 6px + p^2 = 0$

$\Rightarrow 10x^2 + 6x - 52 = 0$

Now we have to solve this equation. The key step here is to recognise 2 aspects.

1) there is a factor of 2

2) We already know that $(x-2)$ is a factor. Why?

$10x^2 + 6x - 52 = 0$

$\Rightarrow 2(5x^2 + 3x - 26) = 0$

$\Rightarrow 2(x-2)(5x+13) = 0$

so $x = 2$ which is point A

or $x = \frac{13}{5} = 2.6$ which is point B.

back to ① $y = -3 \times 2.6 + 1$

$= -6.8$

so B is $(2.6, -6.8)$

Suppose we hadn't spotted the two "tricks" to factorise

$10x^2 + 6x - 52$

well let's try Lizzie's Method

$\Rightarrow x^2 + 6x - 520$

factors 520 "close" to each other.

$520 = 26 \times 20$

$\Rightarrow (x-20)(x+26)$

$\Rightarrow (x - \frac{20}{10})(x + \frac{26}{10})$

$\Rightarrow (x-2)(5x+13)/2$

or even just $(x-2)(10x+26)$

and we now have that other x value $13/5$

Very tricky question.

28) For $x < 3$

the gradient is negative.

END OF EXAM

I think that Qu 27 proved so tricky that Qu 28 got curtailed into a Question which is frankly a bit anticlimatic. If a student has managed to work all the way through Qu 27) then this is a doddle.

And it cost me an extra sheet of paper for worked solutions.

This was the hardest Paper 3 I've come across since introduction. Worth checking the examiner's report.

ROBERT GOODHAND