Paper 2 Nov 19

1) $4x^2(3x+5)$
$= 12x^3 + 20x^2$ ✓

2) $1\text{km} \times \frac{1000\text{m}}{1\text{km}} \times \frac{1000\text{mm}}{1\text{m}}$

$1\text{km} = 10^6$ millimetres ✓

3) $\frac{\frac{7}{12} + \frac{3}{4}}{2} = \frac{\frac{7}{12} + \frac{9}{12}}{2}$

$= \frac{8}{12} = \frac{2}{3}$ ✓

4) This is the graph of $y = \frac{1}{x}$
It is a favourite of examiners

5) LCM of 120 and 144

| 120 | 144 |
|---|---|
| 240 | 288 |
| 360 | 432 |
| 480 | 576 |
| 600 | 720 |
| 720 ← | here it is. |

A longer way would be to find HCF

| | |
|---|---|
| 1 x 120 | 1 x 144 |
| 2 x 60 | 2 x 72 |
| 3 x 40 | 3 x 48 |
| 4 x 30 | 4 x 36 |
| 5 x 24 ← | 6 x 24 ← HCF |
| 6 x 20 | 8 x 18 |
| 8 x 15 | 9 x 16 |
| 10 x 12 | 12 x 12 |

So $120 \times 144 = 24 \times \text{LCM}$
$\text{LCM} = 720$ ✓

We wouldn't actually find the LCM this way but it's a useful relationship to know that maths teachers rarely tell you.

6) Correlation is strong positive ✓
Draw line of best fit and read off estimated long jump.
Lines of best fit are best drawn with a clear transparent ruler.

c) high jump of 125 - do not use line of best fit as this is an extrapolation. Never extrapolate.
(However in other questions examiners will ask you to extrapolate !!!)

7. Speed $= \frac{\text{distance}}{\text{time}} = \frac{110}{2}$
$= 55$ mph.

Time $= \frac{\text{distance}}{\text{speed}} = \frac{44}{55}$ hr.
$= \frac{44 \times 60}{55}$ min
$= 48$ min. ✓

8. $a(3x-10) = 21x + 2b$
$3ax - 10a = 21x + 2b$
we can directly equate powers of $x$.
$x^1$. $3a = 21$ so $a = 7$
$x^0$ $-10a = 2b$
$-70 = 2b$
$b = -35$ ✓

9)
$\angle PKN = 56°$
$\angle PKJ = \frac{180 - 56}{2} = 62°$
Bearing J from K $= 360 - 56 - 62$
$= 242$ ✓
I was taught not to put the degree sign after the bearing by examiners seem to like it.
Bearings are always 3 figures
so Bearing K from J is
$242 - 180 = 062$

NB If the bearing A from B is $x$ then the bearing B from A is
$x \pm 180$
whichever answer lies in the range 0-360

10) linear sequence

0 1 2 3 4 5 6 7 ..
-3 1 5 9 13 17 21
(+4)

Having established the jumping number it's safest and easiest to work back and find the 0th term - the "what we get for free"

so $n^{th}$ term $= 4n - 3$

$100^{th}$ term $= 400 - 3$
$= 397$ ✓

if we wanted to do this with a pure calculation then treat the values as coordinates

So we have (5, 17) and (6, 21)
$x_1\ y_1$ $x_2\ y_2$

Expression is

$$\frac{y-17}{21-17} = \frac{x-5}{6-5}$$

$$y - 17 = \frac{4(x-5)}{1}$$

$$y = 4x - 20 + 17$$

$$y = 4x - 3$$

{ remember $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$ }

11) Value after 4 years

$= 120000 \times 1.05^4$
$= 145861$
$= £150,000$ ✓

NB. Go back and look at Qu. 6 where the correct answer is to say "never extrapolate" yet here we have a question asking you to extrapolate.

12) Side isosceles triangle s

$s^2 = 15^2 + 8^2$
$s = 17$

(this is a 8/15/17 ∠ triangle)

so perimeter $= 3 \times 16 + 6 \times 17$
$= 150$ cm ✓

13)

| Time | Freq | MP | Product |
|---|---|---|---|
| $0 \le t < 30$ | 24 | 15 | 360 |
| $30 \le t < 50$ | 76 | 40 | 3040 |
| $50 \le t < 60$ | 52 | 55 | 2860 |
| $60 \le t < 90$ | 48 | 75 | 3600 |
| | 200 | | 9860 |

Est. mean time $= \frac{9860}{200} = 49.3$ min. ✓

| Freq. | Class Width | Freq. Density |
|---|---|---|
| 24 | 30 | 0.8 |
| 76 | 20 | 3.8 |
| 52 | 10 | 5.2 |
| 48 | 30 | 1.6 |

a) freq density x class width will recover the frequency and hence draw the histogram.

Addendum

As we've come this far we might as well complete the analysis.
First we estimate which block contains the lower quartile, median and upper quartile

Lower quartile 50th person block (2)
Median 100th person block (3)
Upper quartile 150th person block (3)

Now we imagine vertical lines a b and c in block (2) and block (3) that divide the area into 4 equal blocks.

so $24 + 3.6a = 50$
$a = 7.2$
so lower quartile is at $30 + 7.2 = 37.2$ min.

and $24 + 76 + 5.2b = 100$
$b = 0$
and median lies at 50 min

finally $24 + 76 + 5.2c = 150$
$c = 9.6$
so upper quartile lies at 59.6 min.

Note mean > median so histogram has a slight negative skew.

14) Pressure = force/area

Area = Area trapezium + Area triangle.

$= \frac{1}{2}(10+13)\times 12 + \frac{1}{2}\,10\times 8$

$= \frac{1}{2}(23\times 12 + 10\times 8)$

$= 178\ cm^2$

Pressure : $25/178$

$= 0.1404\ N/cm^2$ ✓

{ NB. Newtons/cm² is not a recognised unit - more usual to say $N/m^2$ so we'd multiply the answer by $10^4$ to get 1404 $N/m^2$. But no matter we give the examiner what he's asked for }

Ralf's model underestimates the area so pressure is actually less than Ralf's estimate

15) $y = \sqrt{w^3}$

$w^3 = y^2$

$w = y^{2/3}$

$= \sqrt[3]{y^2}$ ✓

which is one of the given options.

16) a% of b = b% of a

a% of b $= \frac{a}{100}\times b$

b% of a $= \frac{b}{100}\times a$

both equal $ab/100$ so proved.

This is an incredibly useful relationship.

50% of 26 is 13

but we'd maybe struggle to say what 26% of 50 is without a calculator?

b) 160% of 40 ≠ 140% of 60

she should have said

160% of 40 = 40% of 160

17)

| lemon : orange : apple | total |
|---|---|
| | 80 |
| ≤ 0.85 | > 0.15 |
| ≤ 68 | > 12 |

so as we need >12 the answer is 13.

P(lemon or apple) < 0.71

No. lemon + No apple < 56.8

$31 + a < 56.8$

$a < 56.8 - 31$

$a < 25.8$

Maximum apple is therefore 25. ✓

It's more intuitive to convert the relative frequencies into numbers and then solve inequalities.

18) Area box is

$x(x+5) - 4\times 10\times 10 = 1000$

so $x^2 + 5x - 1400 = 0$ ✓

$(x+40)(x-35) = 0$ given

so $x = -40$ or $x = 35$

except the question specified $x > 0$

so we discard the negative solution.

$x = 35$ is the only answer to this problem.

N.B. However be very careful when discarding a solution. Paul Dirac discarded the negative solution of a quadratic only to realise later he'd missed the prediction of the existence of the positron - an electron with a positive charge.

19) $y = \sin x$ is a periodic function ✓

remember this and you pick up an easy mark.

b) Let's suppose $y = 4x$

what point lies on $y = 4x + 2$

it must be $(7, 30)$ ✓

21) $n$ is the middle number of 3 consecutive numbers

so we have

$(n-1)(n)(n+1)$

$= (n^2-n)(n+1)$

$= n^3 - n^2 + n^2 - n$

$= n^3 - n$

to which we add $n$ to get $n^3$ which is a cube. ✓

Although proved let's try an example

$3 \times 4 \times 5 = 60$

$60 + 4 = 64 = 4^3$

$0 \times 1 \times 2 = 0$

$0 + 1 = 1 = 1^3$

$1 \times 2 \times 3 = 6$

$6 + 2 = 8 = 2^3$

so strong evidence.

22) In the sketch curve the average speed A to B is represented by the chord AB. ✓

NB. At 'A' level we move the point B closer and closer to A and discover "in the limit" an expression for the instantaneous velocity at A.

23)

| | A | B | C | |
|---|---|---|---|---|
| length | 5 : | 10 : | $x$ | ① |
| surface area | 25 : | 100 : | $x^2$ | ② |

Tim tries to deduce surface area by applying ① when he should be applying ②

b) Now

| | | |
|---|---|---|
| Volume | 125 : 1000 : | $x^3$ |
| Now | 30 | $\frac{125}{8} \times 30$ |
| | 1 | $125/8$ |
| | 125 | $\frac{125 \times 125}{8}$ |

so $x^3 = 125 \times 125/8$

$= 5^3 \times 5^3 / 2^3$

$x = 5 \times 5/2$

$= 12\frac{1}{2}$. ✓

This requires careful thought and a clear layout.

Let's check our answer

Volume A $= 30$

Volume B $= 12\frac{1}{2} \times 7\frac{1}{2} \times 5$

$= 468.75$

And $30 \times \frac{125}{8} = 468.75$ } ✓ checks

24) $-2 \leq x \leq 3$

$9 \leq x+y \leq 11$

so $x$ can be $-2, 1, 0, 1, 2, 3$

$x+y$ can be $9, 10, 11$

so $y$ can be

| | | | | | | |
|---|---|---|---|---|---|---|
| 11 | 10 | 9 | 8 | 7 | 6 | |
| ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | |
| 13 | 12 | 11 | 10 | 9 | 8 | |

| | | | | | | |
|---|---|---|---|---|---|---|
| max value $y-x$ | 13 | 13 | 11 | 9 | 7 | 5 |

so max $(y-x) = 13$ ✓

when $x = -2$ and $y = 13$

and $(x+y) = 11$ which is permitted.

A tricky question requiring careful thought.

25) $\angle CED = 4x$ (alternate segment)

$\angle ECD = \frac{180-4x}{2} = 90-2x$

$\angle ACB = 90 - 2x$

so $y + 90 - x + 90 - 2x = 180$

so $y - 3x = 0$

$y = 3x$. ✓

There are many ways to derive this but above is probably the simplest.

26) $P \propto Q^2$

$1.25 = k_1 0.5^2$

$k_1 = 5$

$Q \propto \frac{1}{R}$

$0.5 = k_2/6$

$k_2 = 3$

so $0.8 = 5Q^2$

$Q^2 = 0.16$

$Q = 0.4$

$0.4 = 3/R$

so $R = 3/0.4$

$= 7\frac{1}{2}$ ✓

Check $P = 5Q^2$

$Q = 3/R$

$P = \frac{5 \times 3 \times 3}{R^2}$

$R^2 = \frac{45}{P}$

$R = \sqrt{45/P}$

Substitute $P = 0.8$

$R = \sqrt{45/0.8}$

$= 7\frac{1}{2}$ as before

27) $x_{n+1} = \sqrt[3]{3x_n + 7}$

if $x_1 = 2$

$x_{n+1} = \sqrt[3]{3 \times 2 + 7} \simeq 2.3$

$x_3 = \sqrt[3]{3 \times 2.3.. + 7} =$

Now we keep the value in the calculator and keep reiterating this expression until the input and output match.

and after about 6 iterations we get $x = 2.426$ ✓

ie. $\sqrt[3]{2.426 \times 3 + 7} = 2.426$

which is a really neat question

ROBERT GOODHAND