

June 2023 Paper 2

- 1) Write 30:12
in the form $n:1$

$$\Rightarrow 2\frac{1}{2}:1 \quad \checkmark$$

- 2) Δ Numbers
6 10 15 21 next?

$$\begin{array}{ccccccc} 6 & 10 & 15 & 21 & 28 & \checkmark \\ 4 & 5 & 6 & 7 & & \end{array}$$

You can use this method
for almost any polynomial
sequence.

- 3) reciprocal of $\frac{4}{7}$

$$\Rightarrow \frac{7}{4} \quad \checkmark$$

For a number this is also
the inverse $(\frac{4}{7})^{-1} = \frac{7}{4}$

which is why we use -1
for the inverse of a function

$$\text{inverse } \sin \theta = \sin^{-1} \theta$$

$$\text{but } (\sin \theta)^{-1} = \frac{1}{\sin \theta}$$

- 4) toy price increase
12.5% is £19.53

let toy price be p

$$p \times 1.125 = 19.53$$

$$p = £17.36 \quad \checkmark$$

that we get an exact
answer is a clue that
we're "correct".

- 5) Tell has

45 10p coins

8x as many 2p coins

5p ? not known

$$\text{total} = £17.70$$

$$45 \times 0.1 + 45 \times 8 \times 0.02 + n \times 0.05 = 17.70$$

$$\text{value 2p coins} = 7.20$$

$$\begin{array}{r} \text{value 5p coins} = 17.70 - \\ 4.50 \\ \hline 7.20 \\ \hline \pounds 6.00 \end{array}$$

Ratio is 7.20 to 6 or 6:5. $\checkmark$

- 6a)

regular octagon
find exterior angle

$$\frac{360}{8} = 45^\circ \quad \checkmark$$

- 6b) if octagon has more sides
exterior angle decreases $\checkmark$

- 7) In a game die is rolled
(it's die not dice!)

and a spinner spun

spinner shows x^2 2x 3x

	1	2	3	4	5	6
2x	2	4	6	(8)	10	12
3x	3	(6)	9	12	15	18
x^2	1	4	9	16	(25)	36

table completed $\checkmark$

7b)

Game won if score ≥ 10

$$P(\text{win}) = \frac{8}{18}$$

$$8 \text{ scores} \geq 10$$

$$= \frac{4}{9} \checkmark$$

7c) game played 711 times

$$E(\text{wins}) = 711 \times \frac{4}{9}$$

$$= 316 \checkmark$$

$E(\sim)$ means expectation

$$8) (a-3)x^2 + 2b = 5x^2 + 12$$

$\equiv$ means identity

this means we can

equates just x^0 x^1 x^2

either side of identity

$$a-3 = 5$$

$$2b = 12$$

$$a = 8 \checkmark \quad b = 6 \checkmark$$

this is an important technique in higher maths.

9) is coordinate geometry
print off.

10) is a transformation
print off.

11) hemispherical bowl
radius = 12cm.

water is poured in at rate
 $325 \text{ cm}^3/\text{sec}$ for 8 sec.

if bowl more than 70% full.

$$\text{volume bowl} = \frac{1}{2} \frac{4}{3} \pi 12^3$$

$$= 3619 \text{ cm}^3$$

$$\text{water is } 2600 \text{ cm}^3$$

$$\frac{2600}{3619} = 0.72$$

so more than 70% full. $\checkmark$

12)



similar?

$$\frac{5}{12} = \frac{8}{19.2} = 0.416$$

or

$$\frac{5}{12} - \frac{8}{19.2}$$

$$= \frac{5 \times 19.2 - 12 \times 8}{12 \times 19.2} = 0$$

so similar $\checkmark$

$$13) x \frac{\text{boxes}}{\text{hour}} \times \frac{80 \text{ tea bags}}{1 \text{ box}} \times \frac{1 \text{ hour}}{60 \text{ min}}$$

Note careful use of units

$$\Rightarrow \frac{80x}{60} = \frac{4x}{3} \checkmark$$

Q.E.D.

14) 123 employees

$$10 \leq p < 14 \quad 66$$

$$14 \leq p < 20 \quad 32$$

$$20 \leq p < 40 \quad 15$$

$$40 \leq p < 100 \quad 10$$

Statement A

over 30% $p \geq 17$

$$\frac{16+15+10}{123} = \frac{1}{3}$$

hence could be TRUE

statement B

average hourly rate > 20

estimate wage

$$\begin{aligned} & (66 \times 12 + 32 \times 16 \\ & + 30 \times 15 + 70 \times 10) \\ & \div 123 \end{aligned}$$

c) = £19.95

b) this is an estimate but evidence statement may not be true.

d) mean is not the best average. There may be just one highly paid employee. Better measure of central tendency is MEDIAN

15) $(x^2 - 9xy)(2x + 5y)$

$$\begin{aligned} & = 2x^3 - 18x^2y + 5yx^2 \\ & \quad - 45xy^2 \end{aligned}$$

16) line A $y = ax - 1$
(7, 13)

line B $5y - 3x = 4$

rearrange $y = \frac{3x+4}{5}$

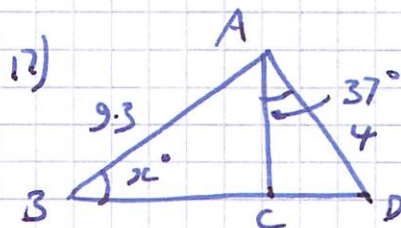
line A $13 = 7a - 1$

$$a = 2$$

gradient A is 2

gradient B is $\frac{3}{5}$

so $A_{\text{grad}} < B_{\text{grad}}$ ✓



$$\cos 37^\circ = \frac{AC}{4}$$

$$AC = 4 \cos 37^\circ$$

$$\sin x^\circ = \frac{4 \cos 37^\circ}{9.3}$$

$$x^\circ = \sin^{-1} \frac{4 \cos 37^\circ}{9.3}$$

check calculator shows

$$\sin 30^\circ = 0.5$$

$$x = 20.0^\circ \quad \checkmark$$

18) $y = \frac{x+8}{x}$ make x subject

$$x+8 = yx$$

$$x - yx = -8$$

$$x(1-y) = -8$$

$$x = \frac{-8}{1-y}$$

$$x = \frac{8}{y-1}$$

19)
$$\begin{array}{cccc} 3 & 20 & 47 & 84 \\ & n^{\text{th}} \text{ term} \end{array}$$

$$\begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & & \\ -4 & 3 & 20 & 47 & 84 & & \\ 7 & 17 & 27 & 37 & & & \\ & 10 & 10 & & & & \end{array}$$

Now this is a standard method. LEARN IT

$$f(n) = an^2 + bn + c$$

c is the zeroth term

$$c = -4$$

a is 2nd line $\div 2!$

$$a = 5$$

1st term is $a+b+c$

$$3 = -4 + 5 + b$$

$$b = 3 + 4 - 5$$

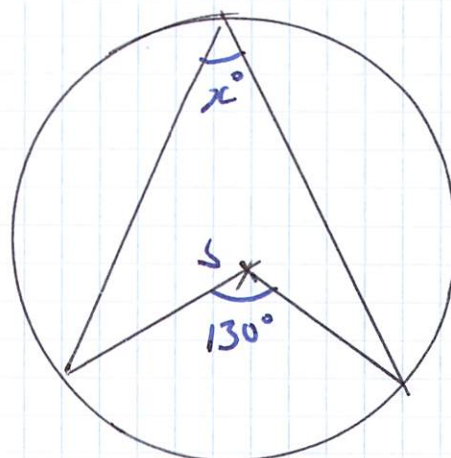
$$b = 2$$

$$f(n) = 5n^2 + 2n - 4 \checkmark$$

$$\begin{aligned} f(4) &= 5 \times 16 + 2 \times 4 - 4 \\ &= 84 \text{ checks} \end{aligned}$$

20 a)

if we assume S is centre of circle

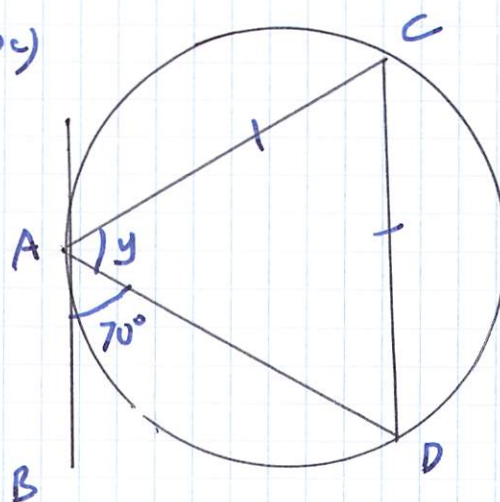


$$x = \frac{130}{2} = 65^\circ \checkmark$$

b)

It is greater than answer to a)

20 y)



Simon's working is incorrect
 $\angle CDA$ is not the alternate segment theorem

$$\angle ACD = 70^\circ$$

$$\angle CAD = \frac{180 - 70}{2}$$

$$\begin{aligned} &= \frac{110}{2} \\ &= 55^\circ \checkmark \end{aligned}$$

- 21) £500 invested
for 3 years
to yield at least £560
after 3 years

$$500 \times x^3 = 560$$

$$x = \sqrt[3]{\frac{560}{500}} \\ = 1.03849$$

so compound interest
must be at least
3.9%

22)

$$x_{n+1} = \sqrt[3]{5(x_n)^2 - 2x_n - 3}$$

if $x = 4$

calculate x_2 and x_3

$$x_2 = \sqrt[3]{5(4)^2 - 2 \times 4 - 3} \\ = 4.102$$

$$x_3 = 4.178$$

$$x_4 = 4.234$$

$$x_5 = 4.275 > 4.25$$

Hint

Don't keep reentering
successive values.

Judicious use of Ans
button and $\sqrt[3]{\quad}$ button.

23)

Set A 1 1 3 5 5 5 6 8

Set B 1 2 4 6 8 8 9

Set C 3 4 5 6

Optim 1 pick 2 cards from A

Optim 2 pick 1 card B
1 card C

To win total = 10

Optim 1 possibilities
5 then 5

Optim 2 possibilities

4 then 6

6 then 4

$$P(\text{win})_1 = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56} \checkmark$$

$$P(\text{win})_2 = \frac{1}{7} \times \frac{1}{4} + \frac{1}{7} \times \frac{1}{4} \\ = \frac{2}{28} = \frac{4}{56} \checkmark$$

Optim 1 is better ✓

24) $a = 65$ nearest integer
 $b = 30$ 1 sig. fig.

upper bound $2a^2 - b^2$

$$2(65.5)^2 - 25^2 \\ = 7955.5 \checkmark$$

25) Simplify

$$\frac{x-5}{x-2} + \frac{x+5}{x+2}$$

* through by $(x-2)(x+2)$

$$\Rightarrow \frac{(x-5)(x+2) + (x+5)(x-2)}{x^2-4}$$

$$= \frac{x^2 - 3x - 10 + x^2 + 3x - 10}{x^2 - 4}$$

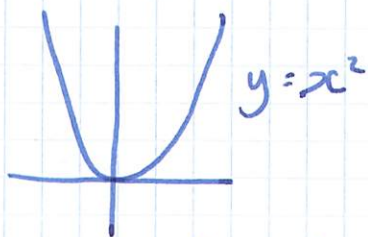
$$= \frac{2x^2 - 20}{x^2 - 4}$$

c) $y = x^2$
transformed to
 $y = (x+3)^2$

this moves curve
three units to left.

$$\begin{bmatrix} -3 \\ 0 \end{bmatrix} \checkmark$$

26)



min. $y = x^2$ $(0, 0)$

min. coordinates

$$y = x^2 + 2$$

curve moved up 2
(coord. moves down 2!)

min. coordinates $(0, 2)$ ✓

d) curve is reflected in
x-axis

$$y = -x^2 \quad \checkmark$$