Maclaurin Series

Assume a function can be expressed as an infinite algebraic series.

$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 \ldots$

setting $x = 0$ gives $f(0) = a_0$ $\qquad a_0 = f(0)$

(to fit the subsequent pattern this is strictly f (0) / 0!)

For a linear function this is the "what you get for free" number

$f'(x) = a_1 + 2 a_2 x + 3 a_3 x^2 + 4 a_4 x^3 + 5 a_5 x^4 \ldots$

setting $x = 0$ gives $f'(0) = a_1$ $\qquad a_1 = f'(0) / 1!$

For a linear function this is "jumping" number

$f''(x) = 2.1\, a_2 + 3.2\, a_3 x + 4.3\, a_4 x^2 + 5.4\, a_5 x^3 \ldots$

setting $x = 0$ gives $f''(0) = 2 a_2$ $\qquad a_2 = f''(0) / 2!$

$f'''(x) = 3.2.1\, a_3 + 4.3.2\, a_4 x + 5.4.3\, a_5 x^2 \ldots$

setting $x = 0$ gives $f'''(0) = 3.2\, a_3$ $\qquad a_3 = f'''(0) / 3!$

$f''''(x) = 4.3.2.1\, a_4 + 5.4.3.2\, a_5 x \ldots$

setting $x = 0$ gives $f''''(0) = 4.3.2\, a_4$ $\qquad a_4 = f''''(0) / 4!$

Hence $f(x) = f(0) / 0! + f'(0) / 1! + f''(0) / 2! + f'''(0) / 3! + f''''(0) / 4! \ldots$

Example

$f(x) = \sin x,\ f'(x) = \cos x,\ f''(x) = {}^-\sin x,\ f'''(x) = {}^-\cos x,\ f''''(x) = \sin(x)$

so $f(0) = 0,\ f'(0) = 1,\quad f''(0) = 0,\quad f'''(0) = {}^-1,\quad f''''(0) = 0$ etc.

Hence referring back to the assumption that an algebraic series exists

$\sin x = x - x^3 / 3! + x^5 / 5! - x^7 / 7! \ldots$ which is an odd function

and it can similarly be shown that

$\cos x = 1 - x^2 / 2! + x^4 / 4! - x^6 / 6! \ldots$ which is an even function. *rg*

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