

Paper 1 June 2023

1 a) $0.7 \times 0.5 ?$
 $= 0.35$

b) $\frac{5}{6} \div 3 ?$
 $= \frac{5}{6} \times \frac{1}{3} = \frac{5}{18}$

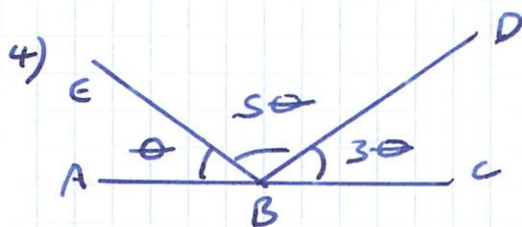
c) $27 \div 0.6$
 $= 27 \times \frac{1}{0.6}$
 $= \frac{270}{6} = 45$
 which "looks" correct.

2) Solve $2x < 26$
 treat $<$ as $=$
 except when $x = -1$
 and swapping left and right.

$$2x < 26$$

$$x < 13$$

3) work out $\left(\frac{3}{2}\right)^2$
 $= \frac{9}{4}$
 as a mixed number
 $= 2\frac{1}{4}$



$$2\theta = 180^\circ$$

$$\theta = 20^\circ$$

$$50 = 100^\circ$$

5) 2 prime numbers
 multiplied together
 even number between
 50 and 60

Ans. $5 \times 11 = 55$

Hint. $7 \times 7 = 49$ so
 try one up and one down.

6) A and B share money
 $5:6$. Bruce gets £96

A : B
 $\times 16 \left(\begin{array}{l} 5 : 6 \\ 80 : 96 \end{array} \right) \times 16$

A gives C $\frac{1}{4}$
 B gives C $\frac{2}{3}$ lucky Carl?

Carl gets $\frac{1}{4} \times 80 + \frac{2}{3} \times 96$
 $= 20 + 64$
 $= £84$

Presumably Carl is the school bully

7) $2^a \times 3 \times 5^2 = 600$

find a

$$2^a = \frac{600}{25 \times 3}$$

$$= 8$$

Now $2^3 = 8$

so $a = 3$

8) Expand

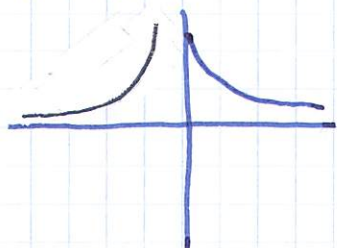
$$5(3x+4) - 2(x-1)$$

$$= 15x + 20 - 2x + 2$$

↑
watch out!

$$= 13x + 22$$

9) for $y = \frac{1}{x}$
Erika sketches this



Two criticisms.

a) Graph should be symmetrical about the y-axis (not intersecting y-axis)

b) Graph should be symmetrical about $y = -x$

(vertical gap is bigger than horizontal gap l.h.s)

10) Sumita is x years old

Beth is $x-1$

Joel is $2x$

The mean is 5

How old is Joel?

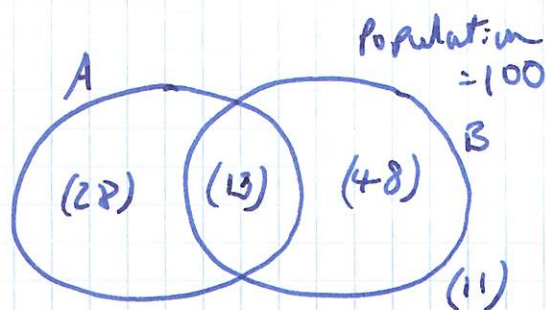
$$5 \times 3 = x + x - 1 + 2x$$

$$15 = 4x - 1$$

$$\text{so } x = 4$$

so Joel is 8

11.



Hint. When number refers to subset put in brackets.

Find $P(A \cap B)$

a) $P(A \cap B) = \frac{13}{100}$

b) $P(A') = \frac{100 - 28 - 13}{100}$

$$= \frac{59}{100}$$

c) $P(A \cup B) = \frac{28 + 13 + 48}{100}$

or more easily $\frac{11}{100}$

12a) $a \times 10^n$
number in standard form

$$? \leq a < ?$$

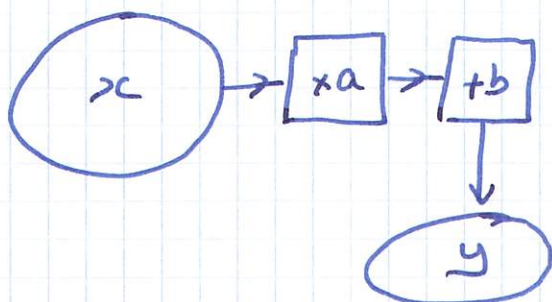
a has to lie between 1 and 10

$$1 \leq a < 10$$

12b) $7200 = 7.2 \times 10^3$

$$7.2 \times 10^{-3} = 0.0072$$

13a) Number machine



Show when input increases
by 2 output increases
by $2a$

$$y = ax + b$$

$$\text{if } x \rightarrow x+2$$

$$y_{\text{new}} = a(x+2) + b$$

$$= ax + 2a + b$$

$$y_{\text{old}} = ax + b$$

$$\text{increase} = \underline{2a}$$

b)

$$f(x) = kx^2$$

$$\text{Kai says } \frac{f(6)}{f(2)} = f(3) \quad \text{becaus}$$

$$\text{Is he correct? } \frac{6}{2} = 3$$

$$f(6) = k36$$

$$f(2) = k4$$

$$\frac{f(6)}{f(2)} = \frac{k36}{k4}$$

$$= 9$$

$$\text{but } f(3) = 9k$$

so Kai is wrong
(but happens to be correct if
 $k=1$ but for wrong reason)

14. 11 whole numbers in numerical order

5 8 LQ 13 19 med. 25 28 UQ ?

$$\text{median} = 2 \times \text{LQ}$$

$$\text{UQ} = 2.5 \times \text{LQ}$$

$$\text{range} = 2 \times \text{IQR}$$

if UQ is to be a whole number
then LQ is EVEN

$$\text{LQ can't be 8 (UQ = 24)}$$

$$\text{LQ can't be 10 (UQ = 25)}$$

$$\text{LQ could be 12 (UQ = 30)}$$

$$\text{so LQ} = 12$$

$$\text{UQ} = 30$$

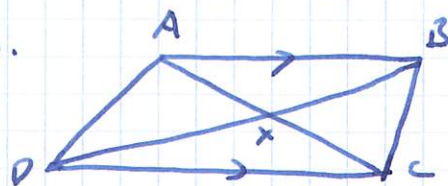
$$\text{IQR} = 18$$

$$\text{range} = 36$$

$$\text{so last number} = 41$$

$$\text{median} = \underline{24}$$

15.



Trapezium. All lengths different

$\triangle AXB$ similar to $\triangle CXD$?

True as angles are the same

$\triangle AXD$ and $\triangle BXC$ congruent?

false

$$\angle ADB = \angle BDC$$

false

Area $\triangle ABC$: Area $\triangle ABD$?

True because same base
and same vertical height

16. Solve

$$2x - 5y = 13$$

$$3x + 4y = 8$$

This is Hopwood's Method

$$\begin{array}{l} \textcircled{1} \quad 2x - 5y = 13 \quad | \times 3 | \quad 6x - 15y = 39 \\ \textcircled{2} \quad 3x + 4y = 8 \quad | \times 2 | \quad 6x + 8y = 16 \end{array}$$

subtract

$$-15y - 8y = 23$$

$$y = -1$$

substitute into $\textcircled{1}$

$$2x + 5 = 13 \quad x = 4$$

check into $\textcircled{2}$

$$12 - 4 = 8 \quad \checkmark \text{ true.}$$

$$\text{so } x = 4 \quad y = -1$$

17.

Solid hemisphere radius x

Solid cylinder radius $3x$
height x

work out

$\textcircled{1}$ surface area hemi : surface area cylinder $\textcircled{2}$

$$\textcircled{1} = \frac{4\pi x^2}{2} + \pi x^2$$

$$\textcircled{2} = 2 \times \pi(3x^2) + 2\pi 3x \times x$$

πx^2 is a common factor

$$\textcircled{1} : \textcircled{2} \text{ is } 2 + 1 : 18 + 6$$

$$3 : 24$$

$$1 : 8$$

$$18. \quad 6 < \sqrt[3]{x} < 7$$

circle possible value x

$$1.9 \quad 20 \quad 45 \quad 290$$

cube everything

$$216 < x < 343$$

$$\text{so } x \text{ is } \underline{290}$$

19.

How many 5-digit odd numbers can be made using these digits once.

$$2 \quad 4 \quad 6 \quad 7 \quad 9$$

$$\text{ending in } 7 \quad 4 \times 3 \times 2 \times 1$$

$$\text{ending in } 9 \quad 4 \times 3 \times 2 \times 1$$

$$\text{total is } \underline{48}$$

20.

$$3K = 4L \quad \textcircled{1}$$

$$K = L + 2M \quad \textcircled{2}$$

$$L = ?M$$

$$\text{from } \textcircled{2} \quad 3K = 3L + 6M$$

$$\text{so } 3L + 6M = 4L$$

$$L = 6M$$

$$21) \quad \text{Express } x^2 - 6x - 15 \text{ in form } (x-a)^2 - b$$

$$(x-3)^2 = x^2 - 6x + 9$$

so we need to subtract 23

$$x^2 - 6x - 15 = (x-3)^2 - 23$$

22) $a = \sqrt{2}$ $b = \sqrt{18}$

so

$$a^2 = 2$$

$$ab = \sqrt{36} = 6$$

$$\frac{b}{a} = \sqrt{18/2} = 3$$

which leaves $a+b$

$$b = \sqrt{18} = 3\sqrt{2}$$

$$\text{so } a+b = 4\sqrt{2}$$

23. $0.\dot{1}3 = 0.1333\dots$

let $x = 0.1333$

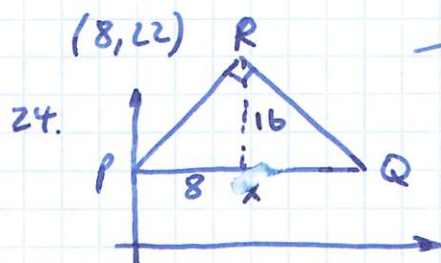
$$10x = 1.3333\dots$$

$$x = 0.1333\dots$$

$$9x = 1.2$$

$$x = \frac{12}{90} = \frac{6}{45}$$

$$= \frac{2}{15}$$



$$\text{Gradient } PR = 2$$

$$\text{so } RX = 16$$

$$\triangle RXP \text{ similar to } \triangle X RQ$$

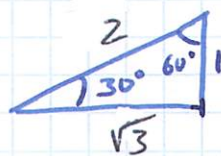
$$\text{so } XQ = 32$$

$$\text{so } Q \text{ is } (40, 8)$$

25) show that

$$\frac{4\sin 30 - \tan 45}{2\cos 30}$$

can be written as $\tan x$



$$\sin 30^\circ = 0.5$$

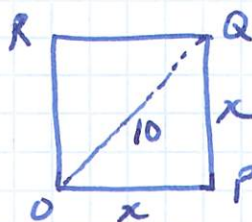
$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\frac{4\sin 30^\circ - \tan 45^\circ}{2\cos 30^\circ}$$

$$= \frac{2-1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} = \tan 30^\circ$$

26.



$$2x^2 = 10^2 \text{ so } x = \sqrt{50}$$

$$\text{Perimeter} = 4\sqrt{50}$$

Now we're given

$$\sqrt{a} : \pi$$

$$\text{so } 4\sqrt{50} : 20\pi$$

$$\frac{\sqrt{50}}{5} : \pi$$

$$\sqrt{a} = \frac{\sqrt{50}}{5}$$

$$a = \frac{50}{25} = 2$$

27.

A journey has 2 stages

	Distance	Average Speed	Time
①	30	a	$\frac{30}{a}$
②	30	b	$\frac{30}{b}$

Show the average speed for whole journey is $\frac{2ab}{a+b}$

Method 1

$$\begin{aligned} \text{Average speed} &= \frac{\text{total distance}}{\text{total time}} \\ &= \frac{60}{\frac{30}{a} + \frac{30}{b}} \end{aligned}$$

Mult. every term by ab

$$\text{Average speed} = \frac{60ab}{30b + 30a}$$

divide every term by 30

$$\text{Average speed} = \frac{2ab}{a+b}$$

Method 2

Average speed is given by the harmonic mean

H of x_1 and x_2 is

$$\frac{2x_1x_2}{x_1+x_2}$$

so solved immediately
(NB. provided distances are equal)