Why Three Cosine Double Angle Formulae but only one Sine?

We have

$\cos 2\theta \quad = 1 - 2\sin^2\theta$

$\qquad\qquad = 2\cos^2\theta - 1$

$\qquad\qquad = \cos^2\theta - \sin^2\theta$

but only $\sin 2\theta = = 2 \sin\theta \ \cos\theta$

Also there is a specific set of the power series where we express

$\cos^n$ in cosines (n odd or even)

$\sin^n$ in cosines (n even)

$\sin^n$ in sines (n odd)

but **<u>not</u>**

$\cos^n$ in sines with n odd or even

$\sin^n$ in sines with n even

$\sin^n$ in cosines with n odd

The same asymmetry is reflected in all hyperbolic relationships.

It also surfaces in the seemingly innocuous

if $\quad \sinh x = \tan y$

then $\ \cosh x = \sec y$

and $\quad \tanh x = \sin y$

Why does the cosh invert?

Because we already know

$\qquad \cosh x = \cos ix$

and the introduction of i gives the clue to the asymmetry.

We have to move everything into the complex plane to see the full picture .Circular functions have period 2π but hyperbolic functions have period $2i\pi$.

In establishing our sin cos relationships we allocate sine to the IMAGINARY (the y) axis.

Now $e^{ix} = \cos x + i \sin x$

let $x = 2A$

$e^{i2A} = \cos 2A + i \sin 2A$

but $e^{i2A} = e^{iA} \times e^{iA}$

$\qquad = (\cos A + i \sin A)^2$

Thus we multiply out and equate real and imaginary parts to reveal

$\cos 2A = \cos^2 A - \sin^2 A$

which we manipulate to produce the other two but

$\sin 2A = 2 \sin A \cos A$

which we're stuck with.

As we have $\qquad \cos ix = \cos x$

and $\qquad\qquad\quad \sinh ix = i \sin x$

whenever we square up the i sin function the i disappears to give us – sin which is Osborne's rule. ✂ *rg*

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