Permutations, Combinations and the Binomial Theorem

Permutations

Let there be 7 distinct objects a, b, c, d , e and f.

We want to choose three of them and the order matters. There are 7 ways to choose the first, 6 to choose the second and 5 ways to choose the third. So there are $7 \times 6 \times 5$ permutations = 210 .

We write $P (n , r) = n! / (n - r)!$ sometimes written ^nP_r .

Combinations

Suppose that the order does not matter. We're choosing three from seven but it's the final set that matters, not the order we choose them.

Consider the selection abc.

That's indistinguishable from acb, bac, bca, cab and cba – six altogether or 3!

So we divide our 210 permutations by 6 to get 35 combinations.

We write

$C (n , r) = n! / (n - r)! \;\; r!$

sometimes written ^nC_r.

Binomial Theorem

If we conduct an experiment with two outcome, P with probability p and Q with probability (p – 1).

We then repeat this experiment n times. Now $(p + q)^n = 1$ no matter how many times we conduct the experiment.

But now let's multiplying out when n = 2, 3, 4 etc. Is there a pattern?

$(p + q)^2 = p^2 + 2pq + q^2$.

$(p + q)^3 =$

$p^3 + 3p^2q + 3pq^2 + q^3$.

$(p + q)^4 =$

$p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$.

$(p + q)^5 =$

$p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$.

The pattern should now be reasonably clear. *(Ask your tutor to explain Pascal's triangle or read it up.)*

For the last expansion the coefficients are

$^5C_0$, $^5C_1$, $^5C_2$, $^5C_3$, $^5C_4$, $^5C_5$

Each term gives the probability of P occurring 1, 2, 3 … times in the n trials. ∞ *Robert Goodhand*

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