Matrix Transformations

Initial Considerations

Take the x-y plot (2 , 1) and consider all the possible transformations.
We permutate the eight possible arrangements of 1 and 2 with + and – in either order. We then consider all possible transformations of a single point in the x-y plane. There are again eight in total. By symmetry the eight permutations must match up to the eight transformations.

Remember +ve rotation anticlockwise.

❶(2 , 1) ⇒ (2 , 1) Identity I
❷ (2 , 1) ⇒ (–2 , 1) reflects in y
❸ (2 , 1) ⇒ (2 , –1) reflects in x
❹ (2 , 1) ⇒ (–2 , –1) rotation 180°
❺(2 , 1) ⇒ (1 , 2) reflects in y = x
❻ (2 , 1) ⇒ (1 , –2) rotation +90°
❼ (2 , 1) ⇒ (–1 , 2) rotation –90°
❽ (2 , 1) ⇒ (–1 , –2) reflects y= –x

Transformation Matrices

Now consider a matrix consisting of two diagonal 1s and two diagonal 0 s again in all possible permutations of positive and negative.

$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$ is the identity matrix. ❶

$\begin{matrix} -1 & 0 \\ 0 & 1 \end{matrix}$ reflects in y ❷

$\begin{matrix} 1 & 0 \\ 0 & -1 \end{matrix}$ reflects in x ❸

$\begin{matrix} -1 & 0 \\ 0 & -1 \end{matrix}$ rotates 180° ❹

$\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}$ reflects in y = x ❺

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$\begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix}$ rotates –90° ❼

$\begin{matrix} 0 & -1 \\ -1 & 0 \end{matrix}$ reflects in y = –x ❽

Determining the Matrix

Start with the Identity Matrix where column 1 is x and column 2 is y. Suppose we want to reflect in the x-axis. Take x as (1 ,0) and determine its new coordinate. For reflection in x it doesn't change. Now take point y (0 , 1) moves to (0 , –1)
So the transformation matrix is $\begin{matrix} 1 & 0 \\ 0 & -1 \end{matrix}$

Try this on any of the transformations.

Robert Goodhand

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