

Bayes Theorem

Suppose we shake one dice. The dice could land One, two ,three , four, five , six. If we shake the dice a large number of times the proportion of say “five” to the total number of throws approaches $1/6$. We therefore say $P(\text{“five”}) = 1/6$.

The probability of a “five” is one in six.

Suppose we shake two die. We can now chart out the possible outcomes.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Now suppose we wanted the probability that the total was greater than 8.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P(\text{total} > 8) = 10/36$ (leave in this form)

Now suppose we wanted the probability that either die showed a “five”

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Remembering that in probability OR means “either or both”

$P(\text{either shows 5}) = 11/36$

Now let’s define $P(A/B)$ as meaning the probability of A happening given that we know that B has already happened.

So if we let A = “total > 8”
and B = “either shows 5”

First consider how many outcomes are both “total > 8” AND “either shows 5”

	1	2	3	4	5	6
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There are only five throws that meet both.

So $P(\text{total} > 8 / \text{either shows 5}) = 5/11$

But $P(\text{either shows 5} / \text{total} > 8) = 5/10$

Bayes Theorem

Note that

$$P(\text{total} > 8 \text{ AND either shows } 5) = 5/36$$

but also equals

$$P(\text{total} > 8 / \text{either shows five}) \times P(\text{either shows } 5)$$

$$5/11 \times 11/36 = 5/36 \text{ as expected.}$$

But we can also turn around

$$P(\text{either shows } 5 \text{ AND total} > 8) = 5/36$$

but also equals

$$P(\text{either shows } 5 / > 8) \times P(\text{total} > 8) \\ 5/10 \times 10/36 = 5/36 \text{ again as expected.}$$

But now we equate the two

$$P(\text{total} > 8 / \text{either shows five}) \times P(\text{either shows } 5) \\ = P(\text{either shows } 5 / \text{total} > 8) \times P(\text{total} > 8)$$

Now to make this clearer let

$$P(\text{over } 8) = P(A)$$

$$P(\text{either } 5) = P(B)$$

so we can rearrange to give

$$P(A/B) = P(B/A) \times P(A) / P(B)$$

which is **Bayes Theorem**

Practical Outcomes

Suppose we said $P(A)$ = probability covid
and $P(B)$ = positive on a test

We want nearly everyone who has covid
to show up on the test so we design

$$P(B/A) = 99/100 \text{ so few slip through.}$$

$$\text{But suppose } P(A) = 1/5$$

$$\text{so } P(B) \cong 25/100$$

$$P(A/B) = P(B/A) \times P(A) / P(B)$$

$$= 99/100 \times 1/5 / 25/100 = 19/99 \cong 20\%$$

That is by making sure the test “catches everyone” who does have covid that doesn’t mean being positive on the test means you really do have covid. There’s a world of difference between testing positive given you really do have covid and you really do have covid given you’ve tested positive. Setting the bar high means a lot of false positives will occur. This phenomena becomes particularly noticeable when you try and test for something that has a relatively low incidence of occurring in the population.

Bayes theorem will show that even when you get a positive result the probability that you actually have the condition is still very low and further tests are required.

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