# Fundamental Theorem of Calculus

## *Introduction*

This sheet explains the fundamental theorem of calculus – that integration is the inverse of differentiation. It's short on mathematical rigour but longer on correct terminology.

## *Differentiation Review*

Let there be a function $y = f(x)$. We wish to know the gradient at any value of x. Assume there is some function $G(x)$ that will give this. Now focus on a very short segment of the plotted curve at-coordinate $x_I$ and a short distance away $x_I+h$. The gradient at $x_I$ is approximately

$G(x_I) \cong \{ f ( x_I + h ) – f ( x_I ) \} / h$

We can determine the exact expression for $G(x)$ by taking the limit as $h \rightarrow 0$

$G(x)$ is called the derivative of $f( x )$ and is written $f'(x)$ – pronounced "f prime x". Finding the derivative is called differentiation.

## *Integration*

Now we repeat the process but now wish to determine the area under the curve $f(x)$ from zero to some point $f(x_I)$. Assume given by some function $A(x)$.

The area is made up of many thin strips each width h and height $f(x)$. Summing all these together is termed integration and the sign is an elongated S.

Area = $\int f(x)\,dx$ where dx is the limit of $h \rightarrow 0$.

Now assume that the derivative of the new function $A(x)$ exists. Therefore

$A'(x) = \{ A(x_I+h) – A(x_I) \} / h$

Now the expression $A(x_I+h) – A(x_I)$ is the area of the thin strip from $x_I$ to $x_I + h$

The area of that thin strip $\cong f(x) \times h$. So

$\{ A(x_I+h) – A(x_I) \} = h\, f(x)$

and $A'(x) = f(x)$.

Therefore the area function must be the antiderivative of $f(x)$ which we write $F(x)$.

## *Fundamental Theorem of Calculus*

The process of finding the area under a curve between a and b is defined as the definite integral of $f(x)$ written $\int_a^b f(x)\,dx$.

It is calculated by finding the antiderivative of $f(x)$ – that is the function that if differentiated gives $f(x)$.

So $\int_a^b f(x)\,dx. = [F(x)]^b_a$

In practice the distinction is blurred and we simply view the symbol $\int dx$ to mean find the antiderivative $F(x)$.

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