Conditional Probability

Given events A and B, is it self-evident that

$P (A \text{ AND } B) = P (A / B) \times P (B)$

That is the probability of A AND B occurring is the probability of A occurring given that B has occurred multiplied by the probability that B occurs.

If A and B are in fact independent then $P (A / B) = P (A)$

Proof?

Assume that the best we can assume is $P (A / B) = k \times P (A \text{ AND } B)$

That is the probability of A occurring given that B has occurred is a linear function of the probability that both A and B occurs – which to me seems a dubious assumption anyway❶. But let's proceed on that basis.

Now set B = A

So $P(A / A)$	$= k \times P (A \text{ AND } A) = k \times P (A)$
But $P (A / A)$	$= 1$
So 1	$= k \times P (A)$
So k	$= 1 / P (A)$ and we deduce
$P(A / B)$	$= P (A \text{ AND } B) / P (A)$ as above

Bayes Theorem

If we assume $P (A \text{ AND } B) = P (B \text{ AND } A)$ – a reasonable assumption❷ - then Bayes Theorem follows immediately

$P (A \text{ AND } B)$	$= P (A / B) \times P (B)$ and
$P (B \text{ AND } A)$	$= P (B / A) \times P(A)$ and equating the two we get
$P (A / B) \times P (B)$	$= P (B / A) \times P (A)$ and rearranging
$P(B / A)$	$= P (A / B) \times P (B) / P (A)$

The probability drawing an Ace given that it's a Heart ($^1/_{13}$) is given by the probability it's a Heart given that it's an Ace ($^1/_4$) multiplied by the ratio of probability drawing an Ace ($^4/_{52}$) over probability drawing a heart ($^1/_4$).

ie $^1/_{13} = {^1/_4} \times {^4/_{52}} / {^1/_4}$ which is true if not particularly enlightening.

Notes

❶ *but that's what I was taught on my postgraduate statistics course*

❷ *which can be proved but is extremely tedious*

♋ *rg*

Conditional Probability

Given events A and B, is it self-evident that

P (A AND B) = P (A / B) × P (B)

That is the probability of A AND B occurring is the probability of A occurring <u>given that</u> B has occurred multiplied by the probability that B occurs.

If A and B are in fact independent then P (A / B) = P (A)

Proof?

Assume that the best we can assume is P (A / B) = k × P (A AND B)

That is the probability of A occurring given that B has occurred is a linear function of the probability that both A and B occurs – which to me seems a dubious assumption anyway❶. But let's proceed on that basis.

Now set B = A

So P(A / A)	= k × P (A AND A) = k × P (A)
But P (A / A)	= 1
So 1	= k × P (A)
So k	= 1 / P (A) and we deduce
P(A / B)	= P (A AND B) / P (A) as above

Bayes Theorem

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P(B /A)	= P (A / B) × P (B) / P (A)

The probability drawing an Ace given that it's a Heart ($^1/_{13}$) is given by the probability it's a Heart given that it's an Ace ($^1/_4$) multiplied by the ratio of probability drawing an Ace ($^4/_{52}$) over probability drawing a heart ($^1/_4$).

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