Moments Iss2

Introduction

Statistical **Moments** play a crucial role when specifying a **probability distribution**. With them we can describe the properties of statistical distribution and help define the distribution.

If the random variable of interest is X ie x_1 x_2 x_3 x_4 etc. … then Moments are defined as the X's expected values - $E(X)$, $E(X^2)$, $E(X^3)$, $E(X^4)$,…, etc.

The four moments in are- the mean, variance, skewness, and kurtosis.

To compare data sets, describe them using these four statistical moments.

The First Moment

The first central moment is the expected value, generally called the average but strictly the **mean**. It is a measure of **central tendency** along with **median** and **mode**.

The definition is the value of μ such that $\sum_{i=1}^{i=n} (x_i - \mu) = 0$

or more simply $\sum (x_i - \mu) = 0$

This calculation has a similarity to calculating the turning moments in a balanced beam – hence the name.

The formula is $\sum x_i / n$.

This is the value μ for a parent population when x_i includes the whole population. If you are analysing a sample set from the parent population, the mean is termed $\bar{x}$, a particularly annoying term to write in Word.

$\bar{x}$ is also the best estimate of μ for the parent population in the absence of any other information about how the data was collected.

Proof the Sum of First Moments about the mean is zero?

It is required that $\sum (x_i - \mu) = 0$

$$\sum (x_i - \mu) = \sum (x_i) - \Sigma(\mu)$$
$$= \sum (x_i) - n\mu$$

Therefore $\mu = {}^1/_n \sum (x_i)$ which is what we would expect.

The Second Moment

The variance of a parent population is defined as $\sigma^2 = \{ \sum (x_i - \mu)^2 \} / n$

The standard deviation is σ , the square root of the variance.

Multiplying out the brackets gives $\{ \sum (x_i^2 - 2\mu x_i + \mu^2) \} / n$

Multiplying through by $\sum$ gives $\{ \sum (x_i)^2 - 2\mu \sum x_i + n\mu^2 \} / n$

Dividing out the last n gives $^1/_n \sum (x_i)^2 - 2\mu\, ^1/_n \sum x_i + \mu^2$

But $^1/_n \sum x_i = \mu$ so that simplifies to $^1/_n \sum (x_i)^2 - 2\mu^2 + \mu^2$

and we finally produce $^1/_n \sum (x_i)^2 - \mu^2$

the **mean** of **squares** minus **square** of **means**.

μ can strictly take any value k but when $k = \mu$ then σ^2 is a minimum.

Skew

$$\gamma = {}^1/_n \sum (x_i - \mu)^3 / \sigma$$

For a simpler calculation Pearson defined

Pearson's 1st skewness coefficient = (mean – mode) / standard deviation

Pearson's 2nd skewness coefficient = 3 (mean – median) / standard deviation

Kurtosis

$$\kappa = {}^1/_n \sum (x_i - \mu)^4 / \sigma^2$$

But applying this formula to a sample gives a biased estimate of the parent population. The unbiased formula is more complex.

"Normal" distributions are called mesokurtic and have a kurtosis of 3.

Kurtosis over 3 is termed "excess kurtosis"

Thick tailed distributions are leptokurtic (positive). This is because raising to 4th power emphasises the outliers.

Thin tailed distributions are called platykurtic (negative)

Financial websites are the best place to investigate this further.

Leptokurtic investments are the risky ones because of the possible greater variations.

Platykurtic investments are safe because the returns are low but the probability of heavy gains or losses are low.

Platy is Latin and means flat and wide.

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