Reciprocals and Inverses

Reciprocal

A reciprocal is the multiplicative inverse of a number.

So the reciprocal of 5 is $^1/_5$ which can be written 5^{-1}

The reciprocal of x is $^1/_x$ which can be written as x^{-1}

Therefore $5 \times 5^{-1} = 1$ just as $x \times {}^1/_x = 1$ where x is a number.

The reciprocal of 5x + 3 is $^1/_{(5x+3)}$ which might be written $(5x + 3)^{-1}$ .Avoided because we will be using the same terminology for inverse and it gets confusing.

Inverse

If f(x) = y then $f^{-1}(y) = x$ - the function that will turn the output back into the input.

So the inverse of 5 is again $^1/_5$ which can be written 5^{-1}.

The inverse of x is also $^1/_x$ which can be written as x^{-1}.

The inverse of $f(x) = 5x + 3$ is $f^{-1}(x) = {}^1/_5\,(x - 3)$ so $f(x) \times f^{-1}(x) \neq 1$ *(definitely not)*

Note that we are using the same nomenclature " $^{-1}$ " but it does not represent the same mathematical operation. That's why $(5x + 1)^{-1}$ is to be avoided because it isn't explicitly clear whether we are referring to a reciprocal or an inverse.

Summary

Reciprocals are a subset of inverse. All reciprocals are inverses but not all inverses are reciprocals. Because we can represent the inverse of number by x^{-1} we use the symbol $^{-1}$ also to represent inverses. But don't think when you see a $^{-1}$ it necessarily means the reciprocal – it could be an inverse.

Examples

The functions sine (sin x), cosine (cos x) and tan (tan x) are strictly called **circular** functions, because they are derived from study of the circle.

Their reciprocals are cosec x ($^1/_{\sin x}$), sec x ($^1/_{\cos x}$) and cot x ($^1/_{\tan x}$)

Their inverses are $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ – pronounced eg "*sine to the minus 1*"

Enter x into sin x on your calculator to get the defined ratio of $^{\text{opposite}}/_{\text{hypotenuse}}$.

Enter x into $\sin^{-1}x$ to get the ratio and now the output is an angle.

The $^{-1}$ is the symbol used on Japanese calculators but these inverses are NOT reciprocals. Because of the confusion, British mathematicians use the terms arcsin (inverse of sine), arccos (inverse of cosine) and arctan (inverse of tangent) but the author of this paper believes this introduces as many confusing issues as it solves. After all British mathematicians happily use $f^{-1}(x)$ for the inverse of f(x).

This page can be ignored at GCSE though the author thinks it important to prepare students for the bigger picture to come at higher mathematics

Hyperbolic Functions

The **circular** functions have **hyperbolic** counterparts called hyperbolic functions. They are sinh (pronounced shine) cosh (pronounced to rhyme with dosh) and tanh (pronounced tan "h").

The reciprocals are cosech, sech and cotanh with sech pronounced "*shec*".

The 6 inverses are $\sinh^{-1}$, $\cosh^{-1}$, $\tanh^{-1}$, cosech^{-1}, sech^{-1}, and cotanh^{-1}.

Confusingly, British mathematicians prefer the inverses of these to be termed arsinh, arcosh, artanh (*and note well it's "ar" not "arc" for technical reasons*) but try an internet search of "arcsinh", which doesn't exist and see how many hits you get, including Wolfram Mathworld and Wikipedia.

Trigonometric Functions

This means in total there are 24 **trigonometric** functions. There are 3 **circular** functions plus the reciprocals making 6 in all. Every one of these 6 functions have their own inverse making 12. Finally every one of these 12 functions has their own corresponding **hyperbolic** function making 24 in total. They all fit together in one beautiful pattern.

Circular	sin	cos	tan	**Reciprocal**	cosec	sec	tan
Inverse	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$	**Inverse**	cosec^{-1}	$\sec^{-1}$	$\tan^{-1}$
Sometimes termed							
	arcsin	arccos	arctan		arccosec	arcsec	arctan
Hyperbolic	sinh	cosh	tanh	**Reciprocal**	cosech	sech	tanh
Inverse	$\sinh^{-1}$	$\cosh^{-1}$	$\tanh^{-1}$	**Inverse**	cosech^{-1}	sech^{-1}	$\tanh^{-1}$
Sometimes termed							
	arsinh	arcosh	artanh		arcosech	arsech	artanh

Footnote

All these 24 functions always use lower case letters. These functions can be expanded where we replace x or y in sin x sin y with what is termed a complex number which is written Sin w, Sin z where w and z are complex numbers.

All this will be covered in post 16 studies but are essential for subjects such as practical electrics or electronics. *Robert Goodhand*

Reciprocals and Inverses

Reciprocal

A reciprocal is the multiplicative inverse of a number.

So the reciprocal of 5 is $^1/_5$ which can be written 5^{-1}

The reciprocal of x is $^1/_x$ which can be written as x^{-1}

Therefore $5 \times 5^{-1} = 1$ just as $x \times {}^1/_x = 1$ where x is a number.

The reciprocal of $5x + 3$ is $^1/_{(5x+3)}$ which might be written $(5x + 3)^{-1}$ .Avoided because we will be using the same terminology for inverse and it gets confusing.

Inverse

If $f(x) = y$ then $f^{-1}(y) = x$ - the function that will turn the output back into the input.

So the inverse of 5 is again $^1/_5$ which can be written 5^{-1}.

The inverse of x is also $^1/_x$ which can be written as x^{-1}.

The inverse of $f(x) = 5x + 3$ is $f^{-1}(x) = {}^1/_5\,(x - 3)$ so $f(x) \times f^{-1}(x) \neq 1$ *(definitely not)*

Note that we are using the same nomenclature " $^{-1}$ " but it does not represent the same mathematical operation. That's why $(5x + 1)^{-1}$ is to be avoided because it isn't explicitly clear whether we are referring to a reciprocal or an inverse.

Summary

Reciprocals are a subset of inverse. All reciprocals are inverses but not all inverses are reciprocals. Because we can represent the inverse of number by x^{-1} we use the symbol $^{-1}$ also to represent inverses. But don't think when you see a $^{-1}$ it necessarily means the reciprocal – it could be an inverse.

Examples

The functions sine (sin x), cosine (cos x) and tan (tan x) are strictly called **circular** functions, because they are derived from study of the circle.

Their reciprocals are cosec x ($^1/_{\sin x}$), sec x ($^1/_{\cos x}$) and cot x ($^1/_{\tan x}$)

Their inverses are $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ – pronounced eg "*sine to the minus 1*"

Enter x into sin x on your calculator to get the defined ratio of $^{\text{opposite}}/_{\text{hypotenuse}}$.

Enter x into $\sin^{-1}x$ to get the ratio and now the output is an angle.

The $^{-1}$ is the symbol used on Japanese calculators but these inverses are NOT reciprocals. Because of the confusion, British mathematicians use the terms arcsin (inverse of sine), arccos (inverse of cosine) and arctan (inverse of tangent) but the author of this paper believes this introduces as many confusing issues as it solves. After all British mathematicians happily use $f^{-1}(x)$ for the inverse of f(x).

This page can be ignored at GCSE though the author thinks it important to prepare students for the bigger picture to come at higher mathematics

Hyperbolic Functions

The **circular** functions have **hyperbolic** counterparts called hyperbolic functions. They are sinh (pronounced shine) cosh (pronounced to rhyme with dosh) and tanh (pronounced tan "h").

The reciprocals are cosech, sech and cotanh with sech pronounced "*shec*".

The 6 inverses are $\sinh^{-1}$, $\cosh^{-1}$, $\tanh^{-1}$, cosech^{-1}, sech^{-1}, and cotanh^{-1}.

Confusingly, British mathematicians prefer the inverses of these to be termed arsinh, arcosh, artanh (*and note well it's "ar" not "arc" for technical reasons*) but try an internet search of "arcsinh", which doesn't exist and see how many hits you get, including Wolfram Mathworld and Wikipedia.

Trigonometric Functions

This means in total there are 24 **trigonometric** functions. There are 3 **circular** functions plus the reciprocals making 6 in all. Every one of these 6 functions have their own inverse making 12. Finally every one of these 12 functions has their own corresponding **hyperbolic** function making 24 in total. They all fit together in one beautiful pattern.

Circular	sin	cos	tan	**Reciprocal**	cosec	sec	tan
Inverse	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$	**Inverse**	cosec^{-1}	$\sec^{-1}$	$\tan^{-1}$
Sometimes termed							
	arcsin	arccos	arctan		arccosec	arcsec	arctan
Hyperbolic	sinh	cosh	tanh	**Reciprocal**	cosech	sech	tanh
Inverse	$\sinh^{-1}$	$\cosh^{-1}$	$\tanh^{-1}$	**Inverse**	cosech^{-1}	sech^{-1}	$\tanh^{-1}$
Sometimes termed							
	arsinh	arcosh	artanh		arcosech	arsech	artanh

Footnote

All these 24 functions always use lower case letters. These functions can be expanded where we replace x or y in sin x sin y with what is termed a complex number which is written Sin w, Sin z where w and z are complex numbers.

All this will be covered in post 16 studies but are essential for subjects such as practical electrics or electronics. *Robert Goodhand*