Minkowski - Separation in 4 –Dimensional Space-time

Euclidean

In two dimensions the separation is

$$\delta s^2 = \delta x^2 + \delta z^2$$

In three dimensions the separation is $\delta s^2 = \delta x^2 + \delta y^2 + \delta z^2$

In four dimensions the separation is

$$\delta s^2 = \delta x^2 + \delta y^2 + \delta z^2 + \delta u^2$$

Einsteinian Spacetime

Einstein adds the 4th dimension of time. Now we might suppose that

$$\delta s^2 = \delta x^2 + \delta y^2 + \delta z^2 + \delta t^2$$

However the units do not match. We need to multiply "time" by a velocity to convert to a separation. The universal constant velocity is the velocity of light – or rather that light travels a specific velocity designated c. So now we have

$$\delta s^2 = \delta x^2 + \delta y^2 + \delta z^2 + \delta(ct)^2$$

However this is still incorrect. In space-time we give time the opposite sign to space.

$$\delta s^2 = \delta x^2 + \delta y^2 + \delta z^2 - \delta(ct)^2$$

The final convention is that we generally want the separation to be positive so we reverse space and time

$$\delta s^2 = \delta(ct)^2 - \delta x^2 - \delta y^2 - \delta z^2$$

In the limit this becomes

$$ds^2 = d(ct)^2 - dx^2 - dy^2 - dz^2$$

Often this is just written

$$s^2 = (ct)^2 - x^2 - y^2 - z^2 \qquad ❶$$

where it is understood the values are representing **<u>changes</u>** in x y z and t and not absolute values.

Minkowski Spacetime

Minkowski defined proper time τ – the time experienced by the object.

Now he produces in one dimension to keep matters simple

$$v_{st}^2 = v_s^2 + v_t^2$$

Velocity in space-time² is the sum of space² and time² velocities.

Now $v_s = dx / dt$ and $v_t = d\tau / dt$

And he sets c = 1. Now all objects move through space-time at the speed of light, which is "1".

So $1 = (dx / dt)^2 + (d\tau / dt)^2$

Multiplying through by t we produce

$$dt^2 = dx^2 + d\tau^2$$

Rearranging gives $d\tau^2 = dt^2 - dx^2$ ❷

and compare this to our original equation.

Interpretation

When at "rest", **<u>all</u>** our allocation of space-time is used up in time. When we move we borrow some of our time-like existence and use that to move in space. Our separation is always constant. A photon uses **<u>all</u>** its allocation in a space-like existence – it experiences no time.

Robert Goodhand

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