

Characteristics of the Normal Distribution Curve

The function is given as

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

$$= \frac{1}{\sqrt{2\pi}} e^{(\frac{1}{2}x^2)^{-1}}$$

$$f'(x) = \frac{1}{\sqrt{2\pi}} (-1) (e^{\frac{1}{2}x^2})^{(-2)} (e^{\frac{1}{2}x^2})$$

$$(x)$$

$$= -x / \sqrt{2\pi} (e^{\frac{1}{2}x^2})$$

Hence I can see that at $x = 0$

$$f'(x) = 0 \text{ and } f(x) = \frac{1}{\sqrt{2\pi}}$$

Integrating the total area under $e^{-\frac{1}{2}x^2}$ is beyond this short paper but it is $\sqrt{2\pi}$

The factor thus normalises the curve to fix the area as 1 so that the areas of individual strips can be interpreted as a probability.

$$\text{Back to } f'(x) = -x / \sqrt{2\pi} (e^{\frac{1}{2}x^2})$$

$$\text{Let } u = -x \text{ so } du = -1$$

$$\text{Let } v = e^{\frac{1}{2}x^2} \text{ so } dv = e^{\frac{1}{2}x^2} (x)$$

$$\text{so } \sqrt{2\pi} \frac{d^2y}{dx^2} = (vdu - u dv) / v^2$$

$$= \{-e^{(\frac{1}{2}x^2)} + xe^{(\frac{1}{2}x^2)}(x)\} / e^{x^2}$$

$$= x^2 - 1 / e^{(\frac{1}{2}x^2)}$$

So I can immediately see that at

$$x^2 - 1 = 0 \text{ we have } \frac{d^2y}{dx^2} = 0$$

That is $x = \pm 1$

We know this to be correct. The point of inflexion where the second derivative is zero occurs at plus/minus 1 standard deviations.

To summarise

Maximum occurs at $(0, \frac{1}{\sqrt{2\pi}})$ and zero curvature occurs at $(\pm 1, e^{-1/2} / \sqrt{2\pi}) \cong (\pm 1, 0.242)$

A quick check on the graphical calculator confirms this.

✕ rg

Characteristics of the Normal Distribution Curve

The function is given as

$$\begin{aligned}
 f(x) &= 1/\sqrt{2\pi} e^{-1/2 x^2} \\
 &= 1/\sqrt{2\pi} e^{(1/2 x^2)^{-1}} \\
 f'(x) &= 1/\sqrt{2\pi} (-1) (e^{1/2 x^2})^{(-2)} (e^{1/2 x^2}) \\
 &= -x / \sqrt{2\pi} (e^{1/2 x^2})
 \end{aligned}$$

Hence I can see that at $x = 0$

$$f'(x) = 0 \text{ and } f(x) = 1/\sqrt{2\pi}$$

Integrating the total area under $e^{-1/2 x^2}$ is beyond this short paper but it is $\sqrt{2\pi}$

The factor thus normalises the curve to fix the area as 1 so that the areas of individual strips can be interpreted as a probability.

$$\text{Back to } f'(x) = -x / \sqrt{2\pi} (e^{1/2 x^2})$$

$$\text{Let } u = -x \text{ so } du = -1$$

$$\text{Let } v = e^{1/2 x^2} \text{ so } dv = e^{1/2 x^2} (x)$$

$$\begin{aligned}
 \text{so } \sqrt{2\pi} \frac{d^2y}{dx^2} &= (vdu - u dv) / v^2 \\
 &= \{-e^{(1/2 x^2)} + x e^{(1/2 x^2)} (x)\} / e^{x^2} \\
 &= x^2 - 1 / e^{(1/2 x^2)}
 \end{aligned}$$

So I can immediately see that at

$$x^2 - 1 = 0 \text{ we have } \frac{d^2y}{dx^2} = 0$$

$$\text{That is } x = \pm 1$$

We know this to be correct. The point of inflexion where the second derivative is zero occurs at plus/minus 1 standard deviations.

To summarise

Maximum occurs at $(0, 1/\sqrt{2\pi})$
and zero curvature occurs at $(\pm 1, e^{-1/2}/\sqrt{2\pi}) \cong (\pm 1, 0.242)$

A quick check on the graphical calculator confirms this.

✕ rg