Quadratic – Further Investigation

Preliminaries

Suppose we say $5x = 10$.
What is the solution? Clearly $x = 2$
In detail we produce $f(x) = 5x - 10$
and for a solution we set $f(x) = 0$

Similarly for a quadratic we say
$f(x) = ax^2 + bx + c$
and then set $f(x) = 0$

The solution $f(x) = 0$ is given by

$x = \{ -b \pm\sqrt{(b^2 - 4ac)} \} / 2a$

The line of symmetry $-b / 2a$ is called the **directrix**.

$(b^2 - 4ac)$ is called the **discriminant**.

Differentiation

If $y = ax^2 + bx + c$ then
$dy/dx = 2ax + b$.
The minimum value occurs when
$dy/dx = 0$, that is $x = -b / 2a$.

This is, of course, the directrix.

Fundamental Theorem of Algebra

A polynomial equation of degree n has exactly n solutions including repeated roots.

For a quadratic there are exactly two solutions.

Usually the quadratic cuts the x-axis at two positions. These are spaced $\pm\sqrt{(b^2 - 4ac)}$ either side of the directrix.

Changing the Roots

If we increase the value of the constant **c** we lift the quadratic and the two roots will approach each other because $b^2 - 4ac$ is reducing.
When $b^2 = 4ac$ the curve touches the x-axis at a single point and we have repeated roots on the directrix.
This would be the case when say
$(x - r)^2 = 0$ so $x^2 - 2xr + r^2 = 0$
$b^2 = (2)^2$ and $4ac = 4r^2$ which equate.
and the solution is $x = -b / 2a$

Solutions when $b^2 - 4ac$ is negative

Suppose $b^2 - 4ac = -4$.
Some mathematicians will say that
$i = \sqrt{-1}$ and reduce $\sqrt{-4}$ to
$\sqrt{-4} = \sqrt{-1} \times \sqrt{4} = 4i$
While the answer is correct the logic is hopeless. The $\sqrt{}$ symbol is strictly defined as the positive root only and there is no positive root of a negative.
Also $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ only applies when a , b are both positive.
$\sqrt{-4} \times \sqrt{-9} = \sqrt{(-4 \times -9)} = \sqrt{36} = 6$
is INCORRECT

the answer is actually -6.

The only resolution is to say you are allowed to extract the – ve from within the $\sqrt{}$sign and convert it to i.
So we still produce $\sqrt{-4} = 4i$
The solutions of $x^2 + 2x - 2$ are
$-1 + i$ and $-1 - i$
but never write $i = \sqrt{-1}$

Robert Goodhand

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