Equations of Motion

Introduction

A car travelling at speed **u** accelerates for **t** seconds to a new speed **v** covering the distance **s**.
We define the constant acceleration "**a**" as $(v - u) / t$
ie the change in velocity over time.
The units of acceleration are
metres / sec^2

First Equation of Motion

Rearranging the definition of acceleration we obtain

1) $v = u + a t$

If we differentiate with respect to time we get
$dv / dt = \mathbf{a}$ which is the definition.

Second Equation of Motion

Taking the "average" speed as

$\frac{1}{2} (u + v)$

and as speed = distance / time so

2) $s = \frac{1}{2} (u + v) t$

Third Equation of Motion

Now we know $v = u + a t$ so substituting for **v** in equation 2)

$s = \frac{1}{2} (u + u + a t) t$ giving

3) $s = u t + \frac{1}{2} a t^2$

Now we can again differentiate this equation with respect to time so
$ds / dt = u + a t$ and $ds / dt = v$ so

$v = u + a t$ as before

Fourth Equation of Motion

Rearranging 1) into $u = v - a t$
and again substituting into

$s = \frac{1}{2} (u + v) t$ gives

$s = \frac{1}{2} (v - a t + v) t$

4) $s = v t - a t^2$

and again we can differentiate with respect to time to give $u = v - a t$

Fifth Equation of Motion

Rearranging $v = u + a t$ to produce

$t = (v - u) / a$

and substituting into $s = \frac{1}{2} (u + v) t$
gives $s = \frac{1}{2} (v + u) (v - u) / a$
Multiplying out gives

$s = \frac{1}{2} (v^2 - u^2) / a$

and rearranging to

5) $v^2 = u^2 + 2 a s$

Summary

We have five terms **s u v a** and **t** and produce five equations each equation dropping **s u v a** or **t**

1)	$v = u + at$	*(no **s**)*
2)	$s = \frac{1}{2} (u + v) t$	*(no **a**)*
3)	$s = u t + \frac{1}{2} a t^2$	*(no **v**)*
4)	$s = v t - \frac{1}{2} at^2$	*(no **u**)*
5)	$v^2 = u^2 + 2 a s$	*(no **t**)*

Further Exercise

Draw a speed – time graph and see which of these equations can be directly identified on the graph
Hint : areas represent distances

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4)	$s = v t - \frac{1}{2} at^2$	*(no u)*
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