Paper 1 May 2019

1) Triangle A : Triangle B

$$\div 5 \begin{cases} 10 : 15 \\ 2 : 3 \end{cases} \div 5$$

$$\times 3 \begin{cases} 2 : 3 \\ 6 : \underline{9} \end{cases} \times 3$$

2) $\left(1\frac{2}{3}\right)^2 = \left(\frac{5}{3}\right)^2$

$= \frac{25}{9}$

$= 2\frac{7}{9}$

3) Arc length semicircle

$= \frac{1}{2} 2\pi r$

$= \frac{1}{2} 2\pi \, 6$

$= 6\pi$

4) $4.625 = 4\frac{5}{8}$

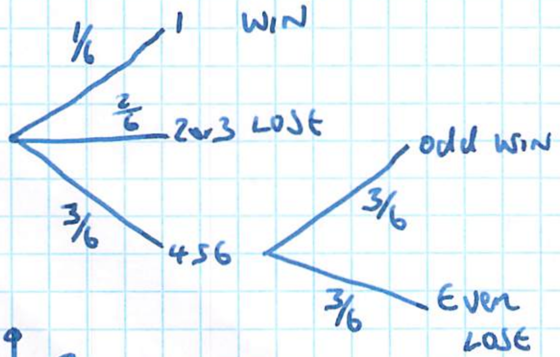
$= \frac{37}{8}$

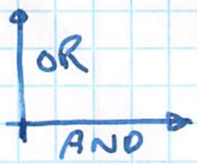
HINT Learn all the eights from $\frac{1}{8}$ + $\frac{7}{8}$ as decimals

5) $0.00097 = 9.7 \times 10^{-4}$ standard form.

5b) see below

6)





Vertical axis ADD to ONE.
Horizontal axis MULTIPLY.

$P(\text{WIN}) = \frac{1}{6} + \frac{3}{6} \times \frac{3}{6} = \frac{15}{36} = \frac{5}{12}$

$P(\text{LOSE}) = \frac{2}{6} + \frac{3}{6} \times \frac{3}{6} = \frac{21}{36} = \frac{7}{12}$

check $\frac{15}{36} + \frac{21}{36} = \frac{36}{36} = 1$ More likely to LOSE

7) Let x be number of people originally at party

$\frac{3}{x} = 0.2$

$x = 3 \times 5 = 15$

so now there are 18 people at party.

Hint. Usually we set the unknown as the final answer but in this case it's easier to set the unknown as the original number.

8) $(3^{12} \div 3^5) \div (3^2 \times 3^1)$

$= 3^7 / 3^3 = 3^4$

$= \underline{81}$

9) Total area $= \pi r^2 = \pi 10^2$

$= 100\pi$

Shaded area $= \frac{1}{2} \pi r^2$

$= \frac{1}{2} \pi 4^2$

$= 8\pi$

Net Unshaded $= 100\pi - 8\pi$

$= 92\pi$

Shaded : Unshaded

$8\pi : 92\pi$

$1 : 11\frac{1}{2}$

10) $n = \frac{60}{t}$ n = items made in hour, t = minutes

In 4 mins machine makes 15 items
In 1 min machine makes 60 items

so we draw a line from (1,60) to (4,15)

Read off from graph

at t = 3min 30sec $n = 17$

Note this is an inverse graph so we need to plot 4 points.

(1,60) (2,30) (3,20) (4,15)

5b) $\frac{3 \times 10^5}{4 \times 10^3} = 0.75 \times 10^2$

$= \underline{75}$

11) Ed : Fay

$\frac{\times 330}{11}$ 7 : 4 11 £330

£210 : £120

-45 $+45$ $\frac{210-120}{2} = 45$

£165 £165

so Ed gives Fay £45.

A clear layout is required for this.

12) Fibonacci series

$-9 \; 2 \; -7 \; -5 \; -12$

fits this pattern.

No others fit this pattern.

13) Volume prism is

$\frac{1}{2} x \times x \times 8.14 = 102$

$x^2 = 204/8.14$

$x^2 \approx 25$

$x \approx 5$

14) $a : b$

$\times 18$ $5 : 3$ $\times 18$ $a = 90$

$90 : 54$

total is 360

$360 - 90 - 54 = 216$

$x : y$

$\times 54$ $1 : 3$ $\times 54$ 4 216

54 162 $\frac{216}{4} = 54$

so we have shown $b = x$

we didn't actually need to work out a or y.

15) $m \leq 10$ $m \leq 20$ $m \leq 30$ $m \leq 40$ $m \leq 50$

20 48 88 108 120

$m \leq 50$ is 120 which checks out.

Plot m against cumulative frequency

Read of $m \leq 15$

cumulative frequency $= 32$

Note each small square is 4 students.

16) Simplify $\frac{4x - 8x^2}{12x - 6}$

$= \frac{4x(1-2x)}{6(2x-1)}$

$= -\frac{4x}{6} = \frac{-2x}{3}$

17) Let the number be y.

So $\frac{1}{2} y (y+3) = y^2$

This is the equation. We are not asked to solve it or even simplify it.

b) $x = \frac{9}{8x}$

$8x^2 = 9$

He errs at the next step

He needs to isolate x before taking the square root.

$x^2 = 9/8$

$x = \pm 3/\sqrt{8}$

18) $x^2 - y^2 \equiv (x+y)(x-y)$

This is called the difference of two squares and hence we use $\equiv$

So $193^2 - 7^2 = (193+7)(193-7)$

$= 200 \times 186$

$= 100 \times 372$

$= 37200$

b) $100a^2 - 81b^2$

$= (10a + 9b)(10a - 9b)$

19) $0.\dot{1}$

strictly $10n = 1.111\ldots$

$n = 0.111\ldots$

$9n = 1$

$n = 1/9$

This pattern should be memorised.

So $0.\dot{1}\dot{3} = 13/99$

$0.\dot{1}3\dot{7} = 137/999$ etc.

20) By alternate segment theorem
theorem $\angle ABC = x^\circ$
so $\angle BAC = x^\circ$
so $\angle ACB = 180 - 2x$

Now $\angle ABC + \angle BCD$
$= x + 180 - 2x + x$
$= x + 180 - x$
$= 180^\circ$
Therefore $AB \parallel CD$.

b) If ABC is equilateral
then $x = 60^\circ$
so $\angle ABC + \angle BCD = 3x$
$= 180^\circ$
so $AB \parallel CD$ still
also AC bisects $\angle BCD$.
This question is a bit tricky.

21)
$2x + 3y = 5p$
$y = 2x + p$

$2x + 3y = 5p$ ①
$-2x + y = p$ ②
so we add ① and ② to get
$4y = 6p$
$y = 1\tfrac{1}{2}p$
from ② $-2x = p - 1\tfrac{1}{2}p$
$-2x = -\tfrac{1}{2}p$
$x = \tfrac{1}{4}p$
and if we put x and y into ①
$2 \times \tfrac{1}{4}p + 3 \times 1\tfrac{1}{2}p = 5p$ checks

22)
$\overrightarrow{CD} = \overrightarrow{CA} + \overrightarrow{AD}$
$= -3b + 6a + 7\tfrac{1}{2}b$
$= 6a + 4\tfrac{1}{2}b$

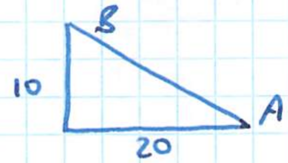
b) for BCD to be straight line
$\overrightarrow{BC}$ $ka + 3b$
$k = 4$
so that $(4a + 3b) \times 1\tfrac{1}{2}$
$= 6a + 4\tfrac{1}{2}b$
Can you see why?

23)
$8^4 \div 32^{2/5}$
$= (2^3)^4 \div (2^5)^{2/5} = 2^{12} \div 2^2$
$= 2^{10}$

24)
$f(x) = \sin(x - 90^\circ)$
$f(0^\circ) = \sin(-90^\circ)$

Now sin is an ODD function.
It has rotational symmetry.
so $\sin 90^\circ = -\sin(-90)$
so $\sin(-90^\circ) = -1$
Alternatively we can say that
$\sin(x - 90^\circ)$ is $\sin 90^\circ$ shifted
right by 90°.

25)
Line OP is perpendicular to BA
Gradient OP is $\frac{\Delta y}{\Delta x} = \frac{8}{4} = 2$
a) So gradient BA is $-\tfrac{1}{2}$
$m_1 m_2 = -1$
Next we find equation of BA
$y - 8 = -\tfrac{1}{2}(x - 4)$
$y - 8 = -\tfrac{1}{2}x + 2$
$y = -\tfrac{1}{2}x + 10$
if $y = 0$ $x = 20$ when $x = 0$ $y = 10$
so we have



by Pythagoras
$BA^2 = 10^2 + 20^2$
$= 500$
$BA = \sqrt{500}$
$= \sqrt{100 \times 5}$
$= 10\sqrt{5}$
This question requires clear thinking and good layout.

26) $y = (x+a)^2 + b$

if the x coordinate turning point is -2 then $a = 2$

$$y = (x+2)^2 + b$$

Now given we have $(3,1)$

$$1 = (3+2)^2 + b$$
$$1 = 25 + b$$
so $b = -24$

so now our equation is

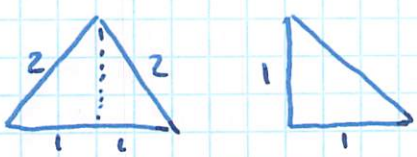
$$y = (x+2)^2 - 24$$

and the y coordinate is -24

If you have access to a graphical calculator it is a good exercise to plot this and identify all the relevant features.

27) $\cos x = \sin 60 \times \tan 30$

Students need to be able to calculate from first principles the sin, cos and tan of $0°$ $30°$ $45°$ $60°$ and $90°$ by drawing 2 standard triangles



In this case $\cos x = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}}$

$$\cos x = \tfrac{1}{2}$$
$$x = \cos^{-1}\left(\tfrac{1}{2}\right)$$
$$= 60°$$

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