Rotating Complex Numbers

Take a piece of graph paper and draw onto it centrally the normal two axis. However label

the x-axis Re (Real) and
the y-axis Im (Imaginary)

Mark off the usual 1,2,3 and negatives as normal.

This is now called an Argand diagram and represents the complex plane.

Plot the point 4 + 3i – that is 4 units along and 3 units up. "4 + 3i" is a complex number with a real part "4" and an imaginary part "3".
Label this ➊.

Now multiply 4 + 3i by "i".

$$(4 + 3i) \times i = 4i + 3i^2 = -3 + 4i$$

Label this ➋.

Now multiply –3 + 4i by "i".

$$(-3 + 4i) \times i = -3i + 4i^2 = -4 - 3i$$

Label this ➌.

Now multiply –4 – 3i by "i".

$$(-4 - 3i) \times i = -4i - 3i^2 = 3 - 4i$$

Label this ➍.

Now check that $(3 - 4i) \times i$

$$= 3i - 4i^2 = 4 + 3i$$

We are now back to our original plot.

Connect ➊with ➌ and ➋with ➍

Note that if plotted correctly the lines are perpendicular and that each multiplication by i rotated a vector by 90° anticlockwise.

This is an example of a simple **conformal mapping**. We actually plotted a vector in the x-y plane and then transformed that vector into the complex plane, performed a rotation by repeatedly multiplying by i and then transformed that back into the x-y plane.

Because the x-y plane and the Re-Im complex plane nominally line up exactly the mapping was relatively easy.

Reflections can also be considered as a rotation. Plot a point, shift the x-axis up to intersect, rotate the point through 180° and then shift the x-axis back again.

Even Translocations can be considered a rotation where the radius is infinite – think of walking 100 miles – are you actually rotating around the earth!

✎ *Robert Goodhand*

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