Differentiating Trig Functions

Circular Functions

$^{d}/_{dx} \sin(m\theta + \phi) = m \cos(m\theta + \phi)$

Now $\sin(\theta + \tfrac{1}{2}\pi)$

$= \sin\theta \cos\tfrac{1}{2}\pi + \cos\theta \sin\tfrac{1}{2}\pi$

$= 0 + \cos\theta$

Therefore $m\cos(m\theta + \phi)$

$= m\sin(m\theta + \phi + \tfrac{1}{2}\pi)$

and $^{d}/_{dx} \sin(m\theta + \phi) =$

$m\sin(m\theta + \phi + \tfrac{1}{2}\pi)$

Differentiating phase shifts angle by $\tfrac{1}{2}\pi$ and introduces the multiplier m.

If we say $f_0 = \sin(m\theta + \phi)$ and represent successive differentiations by f_1 f_2 f_3 etc.

then $f_n = m^n \sin(m\theta + \phi + \tfrac{1}{2}n\pi)$

and if $g_0 = \cos(m\theta + \phi)$

then $g_n = m^n \cos(m\theta + \phi + \tfrac{1}{2}n\pi)$

Function cycles after n = 4 because $4 \times \tfrac{1}{2}\pi = 360°$. Specifically

$^{d}/_{dx} \sin\theta = \cos\theta$ $\;^{d2}/_{dx2} \sin\theta = -\sin\theta$

$^{d3}/_{dx3} \sin\theta = -\cos\theta$ $\;^{d4}/_{dx4} \sin\theta = \sin\theta$

Hyperbolic Functions

$^{d}/_{dx} \sinh(ax + b) = a\cosh(ax + b)$

Now consider $\sinh(a + \tfrac{1}{2}\pi i)$

$\sinh(a + \tfrac{1}{2}\pi i)$

$= \sinh a \cosh\tfrac{1}{2}\pi i + \cosh a \sinh\tfrac{1}{2}\pi i$

$= 0 + i\cosh a$

as $\cosh ix = \cos x$ and $\cos\tfrac{1}{2}\pi = 0$

and $\sinh ix = i\sin x$ and $i\sin\tfrac{1}{2}\pi = i$

Therefore $a\cosh(ax + b)$

$= ai\sinh(ax + b + \tfrac{1}{2}\pi i)$ and

$^{d}/_{dx} \sinh(ax + b)$

$= ia\sinh(ax + b + \tfrac{1}{2}\pi i)$

Differentiating a hyperbolic function phase shifts the hyperbolic angle by $\tfrac{1}{2}\pi i$ and introduces the multiplier ia.

Similarly we will discover

$\sinh(a + i\pi) = -\sinh a$ and

$\cosh(a + i\pi) = -\cosh a$

Thus the periodicity of sinh and cosh is still 2π because in addition we need $i^4 = 1$

Alternatively you add two lots of $i\pi$ to cancel out the negative.

Nevertheless we now have

$^{d}/_{dx} \sinh x = \cosh x$

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because sinh and cosh don't have the asymmetry of sin and cos. ∞ *rg*

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